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# **spreg Documentation**

***Release 1.1.0***

**pysal developers**

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## CONTENTS:

|          |  |          |
|----------|--|----------|
| <b>1</b> | <b>Installation</b>                        | <b>3</b> |
| <b>2</b> | <b>API reference</b>                       | <b>5</b> |
| <b>3</b> | <b>Spatial Regression Models</b>           | <b>7</b> |
| 3.1      | spreg.OLS . . . . .                        | 8        |
| 3.2      | spreg.ML_Lag . . . . .                     | 13       |
| 3.3      | spreg.ML_Error . . . . .                   | 17       |
| 3.4      | spreg.GM_Lag . . . . .                     | 20       |
| 3.5      | spreg.GM_Error . . . . .                   | 25       |
| 3.6      | spreg.GM_Error_Het . . . . .               | 27       |
| 3.7      | spreg.GM_Error_Hom . . . . .               | 30       |
| 3.8      | spreg.GM_Combo . . . . .                   | 33       |
| 3.9      | spreg.GM_Combo_Het . . . . .               | 37       |
| 3.10     | spreg.GM_Combo_Hom . . . . .               | 41       |
| 3.11     | spreg.GM_Endog_Error . . . . .             | 44       |
| 3.12     | spreg.GM_Endog_Error_Het . . . . .         | 47       |
| 3.13     | spreg.GM_Endog_Error_Hom . . . . .         | 51       |
| 3.14     | spreg.TSLS . . . . .                       | 54       |
| 3.15     | spreg.ThreeSLS . . . . .                   | 58       |
| 3.16     | Regimes Models . . . . .                   | 60       |
| 3.16.1   | spreg.OLS_Regimes . . . . .                | 61       |
| 3.16.2   | spreg.ML_Lag_Regimes . . . . .             | 66       |
| 3.16.3   | spreg.ML_Error_Regimes . . . . .           | 70       |
| 3.16.4   | spreg.GM_Lag_Regimes . . . . .             | 75       |
| 3.16.5   | spreg.GM_Error_Regimes . . . . .           | 81       |
| 3.16.6   | spreg.GM_Error_Het_Regimes . . . . .       | 85       |
| 3.16.7   | spreg.GM_Error_Hom_Regimes . . . . .       | 89       |
| 3.16.8   | spreg.GM_Combo_Regimes . . . . .           | 93       |
| 3.16.9   | spreg.GM_Combo_Hom_Regimes . . . . .       | 98       |
| 3.16.10  | spreg.GM_Combo_Het_Regimes . . . . .       | 104      |
| 3.16.11  | spreg.GM_Endog_Error_Regimes . . . . .     | 109      |
| 3.16.12  | spreg.GM_Endog_Error_Hom_Regimes . . . . . | 114      |
| 3.16.13  | spreg.GM_Endog_Error_Het_Regimes . . . . . | 118      |
| 3.17     | Seemingly-Unrelated Regressions . . . . .  | 123      |
| 3.17.1   | spreg.SUR . . . . .                        | 124      |
| 3.17.2   | spreg.SURerrorGM . . . . .                 | 127      |
| 3.17.3   | spreg.SURerrorML . . . . .                 | 130      |
| 3.17.4   | spreg.SURlagIV . . . . .                   | 133      |
| 3.18     | Diagnostics . . . . .                      | 136      |

|          |   |            |
|----------|---|------------|
| 3.18.1   | spreg.diagnostics.f_stat . . . . .          | 137        |
| 3.18.2   | spreg.diagnostics.t_stat . . . . .          | 138        |
| 3.18.3   | spreg.diagnostics.r2 . . . . .              | 139        |
| 3.18.4   | spreg.diagnostics.ar2 . . . . .             | 140        |
| 3.18.5   | spreg.diagnostics.se_betas . . . . .        | 140        |
| 3.18.6   | spreg.diagnostics.log_likelihood . . . . .  | 141        |
| 3.18.7   | spreg.diagnostics.akaike . . . . .          | 142        |
| 3.18.8   | spreg.diagnostics.schwarz . . . . .         | 143        |
| 3.18.9   | spreg.diagnostics.condition_index . . . . . | 144        |
| 3.18.10  | spreg.diagnostics.jarque_bera . . . . .     | 145        |
| 3.18.11  | spreg.diagnostics.breusch_pagan . . . . .   | 146        |
| 3.18.12  | spreg.diagnostics.white . . . . .           | 147        |
| 3.18.13  | spreg.diagnostics.koenker_bassett . . . . . | 149        |
| 3.18.14  | spreg.diagnostics.vif . . . . .             | 150        |
| 3.18.15  | spreg.diagnostics.likratiotest . . . . .    | 151        |
| 3.18.16  | spreg.diagnostics_sp.LMtests . . . . .      | 152        |
| 3.18.17  | spreg.diagnostics_sp.MoranRes . . . . .     | 154        |
| 3.18.18  | spreg.diagnostics_sp.AKtest . . . . .       | 156        |
| 3.18.19  | spreg.diagnostics_sur.sur_setp . . . . .    | 158        |
| 3.18.20  | spreg.diagnostics_sur.sur_lrtest . . . . .  | 158        |
| 3.18.21  | spreg.diagnostics_sur.sur_lmtest . . . . .  | 158        |
| 3.18.22  | spreg.diagnostics_sur.lam_setp . . . . .    | 159        |
| 3.18.23  | spreg.diagnostics_sur.surLMe . . . . .      | 159        |
| 3.18.24  | spreg.diagnostics_sur.surLMlag . . . . .    | 159        |
| <b>4</b> | <b>References</b>                           | <b>161</b> |
|          | <b>Bibliography</b>                         | <b>163</b> |
|          | <b>Index</b>                                | <b>165</b> |

*spreg*, short for “spatial regression,” is a python package to estimate simultaneous autoregressive spatial regression models. These models are useful when modeling processes where observations interact with one another. For more information on these models, consult the Spatial Regression short course by Luc Anselin (Spring, 2017), with the Center for Spatial Data Science at the University of Chicago:

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## INSTALLATION

spreg is installable using the Python Package Manager, *pip*. To install:

```
pip install spreg
```

Further, all of the stable functionality is *also* available in PySAL, the Python Spatial Analysis Library. PySAL can be installed using *pip* or *conda*:

```
pip install pysal #or  
conda install pysal
```





## API REFERENCE



## SPATIAL REGRESSION MODELS

These are the standard spatial regression models supported by the *spreg* package. Each of them contains a significant amount of detail in their docstring discussing how they're used, how they're fit, and how to interpret the results.

|  |  |
|--|--|
| <code>spreg.OLS(y, x[, w, robust, gwk, sig2n_k, ...])</code>   | Ordinary least squares with results and diagnostics.   |
| <code>spreg.ML_Lag(y, x, w[, method, epsilon, ...])</code>     | ML estimation of the spatial lag model with all results and diagnostics; [Ans88]   |
| <code>spreg.ML_Error(y, x, w[, method, epsilon, ...])</code>   | ML estimation of the spatial error model with all results and diagnostics; [Ans88]   |
| <code>spreg.GM_Lag(y, x[, yend, q, w, w_lags, ...])</code>     | Spatial two stage least squares (S2SLS) with results and diagnostics; Anselin (1988) [Ans88]   |
| <code>spreg.GM_Error(y, x, w[, vm, name_y, ...])</code>        | GMM method for a spatial error model, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].                                       |
| <code>spreg.GM_Error_Het(y, x, w[, max_iter, ...])</code>      | GMM method for a spatial error model with heteroskedasticity, with results and diagnostics; based on [ADKP10], following [Ans11].                                  |
| <code>spreg.GM_Error_Hom(y, x, w[, max_iter, ...])</code>      | GMM method for a spatial error model with homoskedasticity, with results and diagnostics; based on Drukker et al.  |
| <code>spreg.GM_Combo(y, x[, yend, q, w, w_lags, ...])</code>   | GMM method for a spatial lag and error model with endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].     |
| <code>spreg.GM_Combo_Het(y, x[, yend, q, w, ...])</code>       | GMM method for a spatial lag and error model with heteroskedasticity and endogenous variables, with results and diagnostics; based on [ADKP10], following [Ans11]. |
| <code>spreg.GM_Combo_Hom(y, x[, yend, q, w, ...])</code>       | GMM method for a spatial lag and error model with homoskedasticity and endogenous variables, with results and diagnostics; based on Drukker et al.                 |
| <code>spreg.GM_Endog_Error(y, x, yend, q, w[, vm, ...])</code> | GMM method for a spatial error model with endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].             |
| <code>spreg.GM_Endog_Error_Het(y, x, yend, q, w[, ...])</code> | GMM method for a spatial error model with heteroskedasticity and endogenous variables, with results and diagnostics; based on [ADKP10], following [Ans11].         |
| <code>spreg.GM_Endog_Error_Hom(y, x, yend, q, w[, ...])</code> | GMM method for a spatial error model with homoskedasticity and endogenous variables, with results and diagnostics; based on Drukker et al.                         |

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Table 1 – continued from previous page

|   |   |
|---|---|
| <code>spreg.TSLS(y, x, yend, q[, w, robust, gw, ...])</code>  | Two stage least squares with results and diagnostics. |
| <code>spreg.ThreeSLS(bigy, bigX, bigyend, bigq[, ...])</code> | User class for 3SLS estimation                        |

## 3.1 spreg.OLS

**class** `spreg.OLS`(*y*, *x*, *w=None*, *robust=None*, *gw=None*, *sig2n\_k=True*, *nonspat\_diag=True*, *spat\_diag=False*, *moran=False*, *white\_test=False*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_w=None*, *name\_gw=None*, *name\_ds=None*)

Ordinary least squares with results and diagnostics.

### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- w** [pysal W object] Spatial weights object (required if running spatial diagnostics)
- robust** [string] If 'white', then a White consistent estimator of the variance-covariance matrix is given. If 'hac', then a HAC consistent estimator of the variance-covariance matrix is given. Default set to None.
- gw** [pysal W object] Kernel spatial weights needed for HAC estimation. Note: matrix must have ones along the main diagonal.
- sig2n\_k** [boolean] If True, then use n-k to estimate  $\sigma^2$ . If False, use n.
- nonspat\_diag** [boolean] If True, then compute non-spatial diagnostics on the regression.
- spat\_diag** [boolean] If True, then compute Lagrange multiplier tests (requires w). Note: see moran for further tests.
- moran** [boolean] If True, compute Moran's I on the residuals. Note: requires spat\_diag=True.
- white\_test** [boolean] If True, compute White's specification robust test. (requires nonspat\_diag=True)
- vm** [boolean] If True, include variance-covariance matrix in summary results
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_gw** [string] Name of kernel weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output

### Examples

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; also, the actual OLS class requires data to be passed in as numpy arrays so the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be an nx1 numpy array.

```
>>> hoval = db.by_col("HOVAL")
>>> y = np.array(hoval)
>>> y.shape = (len(hoval), 1)
```

Extract CRIME (crime) and INC (income) vectors from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). spreg.OLS adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("CRIME"))
>>> X = np.array(X).T
```

The minimum parameters needed to run an ordinary least squares regression are the two numpy arrays containing the independent variable and dependent variables respectively. To make the printed results more meaningful, the user can pass in explicit names for the variables used; this is optional.

```
>>> ols = OLS(y, X, name_y='home value', name_x=['income', 'crime'], name_ds=
↳ 'columbus', white_test=True)
```

spreg.OLS computes the regression coefficients and their standard errors, t-stats and p-values. It also computes a large battery of diagnostics on the regression. In this example we compute the white test which by default isn't ('white\_test=True'). All of these results can be independently accessed as attributes of the regression object created by running spreg.OLS. They can also be accessed at one time by printing the summary attribute of the regression object. In the example below, the parameter on crime is -0.4849, with a t-statistic of -2.6544 and p-value of 0.01087.

```
>>> ols.betas
array([[ 46.42818268],
       [  0.62898397],
       [-0.48488854]])
>>> print round(ols.t_stat[2][0], 3)
-2.654
>>> print round(ols.t_stat[2][1], 3)
0.011
>>> print round(ols.r2, 3)
0.35
```

Or we can easily obtain a full summary of all the results nicely formatted and ready to be printed:

```
>>> print ols.summary
REGRESSION
-----
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES
-----
Data set          :    columbus
Dependent Variable :    home value
↳ 49
Mean dependent var :    38.4362
↳ 3
S.D. dependent var :    18.4661
↳ 46
Number of Observations:
Number of Variables  :
Degrees of Freedom   :
```

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```

R-squared          :      0.3495
Adjusted R-squared :      0.3212
Sum squared residual: 10647.015      F-statistic          :      12.
↪3582
Sigma-square       :      231.457      Prob(F-statistic)    :      5.064e-
↪05
S.E. of regression :      15.214      Log likelihood       :      -201.
↪368
Sigma-square ML    :      217.286      Akaike info criterion :      408.
↪735
S.E of regression ML:      14.7406      Schwarz criterion    :      414.
↪411
-----
↪--
Variable          Coefficient      Std.Error      t-Statistic      _
↪Probability
-----
↪--
CONSTANT          46.4281827      13.1917570      3.5194844      0.
↪0009867
crime             -0.4848885      0.1826729      -2.6544086      0.
↪0108745
income            0.6289840      0.5359104      1.1736736      0.
↪2465669
-----
↪--

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER      12.538

TEST ON NORMALITY OF ERRORS
TEST      DF      VALUE      PROB
Jarque-Bera      2      39.706      0.0000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST      DF      VALUE      PROB
Breusch-Pagan test      2      5.767      0.0559
Koenker-Bassett test    2      2.270      0.3214

SPECIFICATION ROBUST TEST
TEST      DF      VALUE      PROB
White      5      2.906      0.7145
===== END OF REPORT_
↪=====

```

If the optional parameters `w` and `spat_diag` are passed to `spreg.OLS`, spatial diagnostics will also be computed for the regression. These include Lagrange multiplier tests and Moran's  $I$  of the residuals. The `w` parameter is a PySAL spatial weights matrix. In this example, `w` is built directly from the shapefile `columbus.shp`, but `w` can also be read in from a GAL or GWT file. In this case a rook contiguity weights matrix is built, but PySAL also offers queen contiguity, distance weights and `k` nearest neighbor weights among others. In the example, the Moran's  $I$  of the residuals is 0.204 with a standardized value of 2.592 and a p-value of 0.0095.

```

>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))

```

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```

>>> ols = OLS(y, X, w, spat_diag=True, moran=True, name_y='home value', name_x=[
↳ 'income', 'crime'], name_ds='columbus')
>>> ols.betas
array([[ 46.42818268],
       [  0.62898397],
       [-0.48488854]])
>>> print round(ols.moran_res[0],3)
0.204
>>> print round(ols.moran_res[1],3)
2.592
>>> print round(ols.moran_res[2],4)
0.0095

```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**robust** [string] Adjustment for robust standard errors

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**r2** [float] R squared

**ar2** [float] Adjusted R squared

**utu** [float] Sum of squared residuals

**sig2** [float] Sigma squared used in computations

**sig2ML** [float] Sigma squared (maximum likelihood)

**f\_stat** [tuple] Statistic (float), p-value (float)

**logll** [float] Log likelihood

**aic** [float] Akaike information criterion

**schwarz** [float] Schwarz information criterion

**std\_err** [array] 1xk array of standard errors of the betas

**t\_stat** [list of tuples] t statistic; each tuple contains the pair (statistic, p-value), where each is a float

**mulColli** [float] Multicollinearity condition number

**jarque\_bera** [dictionary] ‘jb’: Jarque-Bera statistic (float); ‘pvalue’: p-value (float); ‘df’: degrees of freedom (int)

**breusch\_pagan** [dictionary] ‘bp’: Breusch-Pagan statistic (float); ‘pvalue’: p-value (float); ‘df’: degrees of freedom (int)

**koenker\_bassett** [dictionary] ‘kb’: Koenker-Bassett statistic (float); ‘pvalue’: p-value (float); ‘df’: degrees of freedom (int)

**white** [dictionary] ‘wh’: White statistic (float); ‘pvalue’: p-value (float); ‘df’: degrees of freedom (int)

**lm\_error** [tuple] Lagrange multiplier test for spatial error model; tuple contains the pair (statistic, p-value), where each is a float

**lm\_lag** [tuple] Lagrange multiplier test for spatial lag model; tuple contains the pair (statistic, p-value), where each is a float

**rlm\_error** [tuple] Robust lagrange multiplier test for spatial error model; tuple contains the pair (statistic, p-value), where each is a float

**rlm\_lag** [tuple] Robust lagrange multiplier test for spatial lag model; tuple contains the pair (statistic, p-value), where each is a float

**lm\_sarma** [tuple] Lagrange multiplier test for spatial SARMA model; tuple contains the pair (statistic, p-value), where each is a float

**moran\_res** [tuple] Moran’s I for the residuals; tuple containing the triple (Moran’s I, standardized Moran’s I, p-value)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**sig2n** [float] Sigma squared (computed with n in the denominator)

**sig2n\_k** [float] Sigma squared (computed with n-k in the denominator)

**xtx** [float]  $X'X$

**xtxi** [float]  $(X'X)^{-1}$

**\_\_init\_\_** (*self*, *y*, *x*, *w=None*, *robust=None*, *gwk=None*, *sig2n\_k=True*, *nonspat\_diag=True*, *spat\_diag=False*, *moran=False*, *white\_test=False*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_w=None*, *name\_gwk=None*, *name\_ds=None*)

Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <b>__init__</b> ( <i>self</i> , <i>y</i> , <i>x</i> [, <i>w</i> , <i>robust</i> , <i>gwk</i> , ...]) | Initialize self. |
|--|------------------|

---

## Attributes



|         |
|---------|
| mean_y  |
| sig2n   |
| sig2n_k |
| std_y   |
| utu     |
| vm      |

## 3.2 spreg.ML\_Lag

**class** `spreg.ML_Lag` (*y, x, w, method='full', epsilon=1e-07, spat\_diag=False, vm=False, name\_y=None, name\_x=None, name\_w=None, name\_ds=None*)

ML estimation of the spatial lag model with all results and diagnostics; [Ans88]

### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- w** [pysal W object] Spatial weights object
- method** [string] if 'full', brute force calculation (full matrix expressions) if 'ord', Ord eigen-value method
- epsilon** [float] tolerance criterion in minimize\_scalar function and inverse\_product
- spat\_diag** [boolean] if True, include spatial diagnostics
- vm** [boolean] if True, include variance-covariance matrix in summary results
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output

### Examples

```
>>> import numpy as np
>>> import libpysal
>>> db = libpysal.io.open(libpysal.examples.get_path("baltim.dbf"), 'r')
>>> ds_name = "baltim.dbf"
>>> y_name = "PRICE"
>>> y = np.array(db.by_col(y_name)).T
>>> y.shape = (len(y), 1)
>>> x_names = ["NROOM", "NBATH", "PATIO", "FIREPL", "AC", "GAR", "AGE", "LOTSZ", "SQFT"]
>>> x = np.array([db.by_col(var) for var in x_names]).T
>>> ww = ps.open(ps.examples.get_path("baltim_q.gal"))
>>> w = ww.read()
>>> ww.close()
>>> w_name = "baltim_q.gal"
>>> w.transform = 'r'
>>> mllag = ML_Lag(y, x, w, name_y=y_name, name_x=x_names, name_w=w_name,
    ↪ name_ds=ds_name)
>>> np.around(mllag.betas, decimals=4)
```

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```

array([[ 4.3675],
       [ 0.7502],
       [ 5.6116],
       [ 7.0497],
       [ 7.7246],
       [ 6.1231],
       [ 4.6375],
       [-0.1107],
       [ 0.0679],
       [ 0.0794],
       [ 0.4259]])
>>> "{0:.6f}".format(mllag.rho)
'0.425885'
>>> "{0:.6f}".format(mllag.mean_y)
'44.307180'
>>> "{0:.6f}".format(mllag.std_y)
'23.606077'
>>> np.around(np.diag(mllag.vml), decimals=4)
array([[ 23.8716,   1.1222,   3.0593,   7.3416,   5.6695,   5.4698,
         2.8684,   0.0026,   0.0002,   0.0266,   0.0032,  220.1292])
>>> np.around(np.diag(mllag.v), decimals=4)
array([[ 23.8716,   1.1222,   3.0593,   7.3416,   5.6695,   5.4698,
         2.8684,   0.0026,   0.0002,   0.0266,   0.0032])
>>> "{0:.6f}".format(mllag.sig2)
'151.458698'
>>> "{0:.6f}".format(mllag.logll)
'-832.937174'
>>> "{0:.6f}".format(mllag.aic)
'1687.874348'
>>> "{0:.6f}".format(mllag.schwarz)
'1724.744787'
>>> "{0:.6f}".format(mllag.pr2)
'0.727081'
>>> "{0:.4f}".format(mllag.pr2_e)
'0.7062'
>>> "{0:.4f}".format(mllag.utu)
'31957.7853'
>>> np.around(mllag.std_err, decimals=4)
array([[ 4.8859,   1.0593,   1.7491,   2.7095,   2.3811,   2.3388,   1.6936,
         0.0508,   0.0146,   0.1631,   0.057 ]])
>>> np.around(mllag.z_stat, decimals=4)
array([[ 0.8939,   0.3714],
       [ 0.7082,   0.4788],
       [ 3.2083,   0.0013],
       [ 2.6018,   0.0093],
       [ 3.2442,   0.0012],
       [ 2.6181,   0.0088],
       [ 2.7382,   0.0062],
       [-2.178 ,   0.0294],
       [ 4.6487,   0.    ],
       [ 0.4866,   0.6266],
       [ 7.4775,   0.    ]])
>>> mllag.name_y
'PRICE'
>>> mllag.name_x
['CONSTANT', 'NROOM', 'NBATH', 'PATIO', 'FIREPL', 'AC', 'GAR', 'AGE', 'LOTSZ',
 → 'SQFT', 'W_PRICE']

```

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```

>>> mllag.name_w
'baltim_q.gal'
>>> mllag.name_ds
'baltim.dbf'
>>> mllag.title
'MAXIMUM LIKELIHOOD SPATIAL LAG (METHOD = FULL)'
>>> mllag = ML_Lag(y,x,w,method='ord',name_y=y_name,name_x=x_names,
↳name_w=w_name,name_ds=ds_name)
>>> np.around(mllag.betas, decimals=4)
array([[ 4.3675],
       [ 0.7502],
       [ 5.6116],
       [ 7.0497],
       [ 7.7246],
       [ 6.1231],
       [ 4.6375],
       [-0.1107],
       [ 0.0679],
       [ 0.0794],
       [ 0.4259]])
>>> "{0:.6f}".format(mllag.rho)
'0.425885'
>>> "{0:.6f}".format(mllag.mean_y)
'44.307180'
>>> "{0:.6f}".format(mllag.std_y)
'23.606077'
>>> np.around(np.diag(mllag.vml), decimals=4)
array([[ 23.8716,   1.1222,   3.0593,   7.3416,   5.6695,   5.4698,
         2.8684,   0.0026,   0.0002,   0.0266,   0.0032,  220.1292])
>>> np.around(np.diag(mllag.v), decimals=4)
array([[ 23.8716,   1.1222,   3.0593,   7.3416,   5.6695,   5.4698,
         2.8684,   0.0026,   0.0002,   0.0266,   0.0032])
>>> "{0:.6f}".format(mllag.sig2)
'151.458698'
>>> "{0:.6f}".format(mllag.logll)
'-832.937174'
>>> "{0:.6f}".format(mllag.aic)
'1687.874348'
>>> "{0:.6f}".format(mllag.schwarz)
'1724.744787'
>>> "{0:.6f}".format(mllag.pr2)
'0.727081'
>>> "{0:.6f}".format(mllag.pr2_e)
'0.706198'
>>> "{0:.4f}".format(mllag.utu)
'31957.7853'
>>> np.around(mllag.std_err, decimals=4)
array([[ 4.8859,   1.0593,   1.7491,   2.7095,   2.3811,   2.3388,   1.6936,
         0.0508,   0.0146,   0.1631,   0.057 ]])
>>> np.around(mllag.z_stat, decimals=4)
array([[ 0.8939,   0.3714],
       [ 0.7082,   0.4788],
       [ 3.2083,   0.0013],
       [ 2.6018,   0.0093],
       [ 3.2442,   0.0012],
       [ 2.6181,   0.0088],
       [ 2.7382,   0.0062],

```

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```

        [-2.178 ,  0.0294],
        [ 4.6487,  0.    ],
        [ 0.4866,  0.6266],
        [ 7.4775,  0.    ]])
>>> mllag.name_y
'PRICE'
>>> mllag.name_x
['CONSTANT', 'NROOM', 'NBATH', 'PATIO', 'FIREPL', 'AC', 'GAR', 'AGE', 'LOTSZ',
↪ 'SQFT', 'W_PRICE']
>>> mllag.name_w
'baltim_q.gal'
>>> mllag.name_ds
'baltim.dbf'
>>> mllag.title
'MAXIMUM LIKELIHOOD SPATIAL LAG (METHOD = ORD)'
```

### Attributes

**betas** [array] (k+1)x1 array of estimated coefficients (rho first)

**rho** [float] estimate of spatial autoregressive coefficient

**u** [array] nx1 array of residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant, excluding the rho)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**method** [string] log Jacobian method if 'full': brute force (full matrix computations)

**epsilon** [float] tolerance criterion used in minimize\_scalar function and inverse\_product

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (k+1 x k+1), all coefficients

**vm1** [array] Variance covariance matrix (k+2 x k+2), includes sig2

**sig2** [float] Sigma squared used in computations

**logll** [float] maximized log-likelihood (including constant terms)

**aic** [float] Akaike information criterion

**schwarz** [float] Schwarz criterion

**predy\_e** [array] predicted values from reduced form

**e\_pred** [array] prediction errors using reduced form predicted values

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form))

**utu** [float] Sum of squared residuals

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**\_\_init\_\_** (*self*, *y*, *x*, *w*, *method*='full', *epsilon*=1e-07, *spat\_diag*=False, *vm*=False, *name\_y*=None, *name\_x*=None, *name\_w*=None, *name\_ds*=None)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

|  |                  |
|--|------------------|
| <b>__init__</b> ( <i>self</i> , <i>y</i> , <i>x</i> , <i>w</i> [, <i>method</i> , <i>epsilon</i> , ...]) | Initialize self. |
|--|------------------|

## Attributes

|         |
|---------|
| mean_y  |
| sig2n   |
| sig2n_k |
| std_y   |
| utu     |
| vm      |

## 3.3 spreg.ML\_Error

**class** `spreg.ML_Error` (*y*, *x*, *w*, *method*='full', *epsilon*=1e-07, *spat\_diag*=False, *vm*=False, *name\_y*=None, *name\_x*=None, *name\_w*=None, *name\_ds*=None)  
ML estimation of the spatial error model with all results and diagnostics; [Ans88]

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**w** [Sparse matrix] Spatial weights sparse matrix

**method** [string] if 'full', brute force calculation (full matrix expressions) if 'ord', Ord eigen-value method if 'LU', LU sparse matrix decomposition

**epsilon** [float] tolerance criterion in minimize\_scalar function and inverse\_product

**spat\_diag** [boolean] if True, include spatial diagnostics (not implemented yet)

**vm** [boolean] if True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> np.set_printoptions(suppress=True) #prevent scientific format
>>> db = libpysal.io.open(examples.get_path("south.dbf"), 'r')
>>> y_name = "HR90"
>>> y = np.array(db.by_col(y_name))
>>> y.shape = (len(y), 1)
>>> x_names = ["RD90", "PS90", "UE90", "DV90"]
>>> x = np.array([db.by_col(var) for var in x_names]).T
>>> ww = libpysal.io.open(libpysal.examples.get_path("south_q.gal"))
>>> w = ww.read()
>>> ww.close()
>>> w_name = "south_q.gal"
>>> w.transform = 'r'
>>> mlerr = ML_Error(y, x, w, name_y=y_name, name_x=x_names, name_w=w_name,
    ↪ name_ds=ds_name)
>>> np.around(mlerr.betas, decimals=4)
array([[ 6.1492],
       [ 4.4024],
       [ 1.7784],
       [-0.3781],
       [ 0.4858],
       [ 0.2991]])
>>> "{0:.4f}".format(mlerr.lam)
'0.2991'
>>> "{0:.4f}".format(mlerr.mean_y)
'9.5493'
>>> "{0:.4f}".format(mlerr.std_y)
'7.0389'
>>> np.around(np.diag(mlerr.vcov), decimals=4)
array([[ 1.0648,  0.0555,  0.0454,  0.0061,  0.0148,  0.0014]])
>>> np.around(mlerr.sig2, decimals=4)
array([[ 32.4069]])
>>> "{0:.4f}".format(mlerr.logll)
'-4471.4071'
>>> "{0:.4f}".format(mlerr.aic)
'8952.8141'
>>> "{0:.4f}".format(mlerr.schwarz)
'8979.0779'
>>> "{0:.4f}".format(mlerr.pr2)
'0.3058'
>>> "{0:.4f}".format(mlerr.utu)
'48534.9148'
>>> np.around(mlerr.std_err, decimals=4)
array([[ 1.0319,  0.2355,  0.2132,  0.0784,  0.1217,  0.0378]])
>>> np.around(mlerr.z_stat, decimals=4)
array([[ 5.9593,  0.      ],
```

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```

    [ 18.6902,  0.    ],
    [  8.3422,  0.    ],
    [-4.8233,  0.    ],
    [  3.9913,  0.0001],
    [  7.9089,  0.    ]])
>>> mlerr.name_y
'HR90'
>>> mlerr.name_x
['CONSTANT', 'RD90', 'PS90', 'UE90', 'DV90', 'lambda']
>>> mlerr.name_w
'south_q.gal'
>>> mlerr.name_ds
'south.dbf'
>>> mlerr.title
'MAXIMUM LIKELIHOOD SPATIAL ERROR (METHOD = FULL) '
```

### Attributes

**betas** [array] (k+1)x1 array of estimated coefficients (rho first)

**lam** [float] estimate of spatial autoregressive coefficient

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant, excluding lambda)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**method** [string] log Jacobian method if 'full': brute force (full matrix computations)

**epsilon** [float] tolerance criterion used in minimize\_scalar function and inverse\_product

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**varb** [array] Variance covariance matrix (k+1 x k+1) - includes var(lambda)

**vm1** [array] variance covariance matrix for lambda, sigma (2 x 2)

**sig2** [float] Sigma squared used in computations

**logll** [float] maximized log-likelihood (including constant terms)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**utu** [float] Sum of squared residuals

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

## Methods

|                  |  |
|------------------|--|
| <b>get_x_lag</b> |  |
|------------------|--|

**\_\_init\_\_**(*self*, *y*, *x*, *w*, *method*='full', *epsilon*=1e-07, *spat\_diag*=False, *vm*=False, *name\_y*=None, *name\_x*=None, *name\_w*=None, *name\_ds*=None)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

|  |                  |
|--|------------------|
| <b>__init__</b> ( <i>self</i> , <i>y</i> , <i>x</i> , <i>w</i> [, <i>method</i> , <i>epsilon</i> , ...]) | Initialize self. |
| <b>get_x_lag</b> ( <i>self</i> , <i>w</i> , <i>regimes_att</i> )   |                  |

## Attributes

|                |
|----------------|
| <b>mean_y</b>  |
| <b>sig2n</b>   |
| <b>sig2n_k</b> |
| <b>std_y</b>   |
| <b>utu</b>     |
| <b>vm</b>      |

## 3.4 spreg.GM\_Lag

**class** spreg.**GM\_Lag**(*y*, *x*, *yend*=None, *q*=None, *w*=None, *w\_lags*=1, *lag\_q*=True, *robust*=None, *gwk*=None, *sig2n\_k*=False, *spat\_diag*=False, *vm*=False, *name\_y*=None, *name\_x*=None, *name\_yend*=None, *name\_q*=None, *name\_w*=None, *name\_gwk*=None, *name\_ds*=None)

Spatial two stage least squares (S2SLS) with results and diagnostics; Anselin (1988) [[Ans88](#)]

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x); cannot be used in combination with h

**w** [pysal W object] Spatial weights object



**w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.

**lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).

**robust** [string] If 'white', then a White consistent estimator of the variance-covariance matrix is given. If 'hac', then a HAC consistent estimator of the variance-covariance matrix is given. Default set to None.

**gwk** [pysal W object] Kernel spatial weights needed for HAC estimation. Note: matrix must have ones along the main diagonal.

**sig2n\_k** [boolean] If True, then use n-k to estimate  $\sigma^2$ . If False, use n.

**spat\_diag** [boolean] If True, then compute Anselin-Kelejian test

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis. Since we will need some tests for our model, we also import the diagnostics module.

```
>>> import numpy as np
>>> import libpysal
>>> import spreg.diagnostics as D
```

Open data on Columbus neighborhood crime (49 areas) using libpysal.io.open(). This is the DBF associated with the Columbus shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("columbus.dbf"), 'r')
```

Extract the HOVAL column (home value) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be an numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) and CRIME (crime rates) vectors from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in, but this can be overridden by passing constant=False.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("CRIME"))
>>> X = np.array(X).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

This class runs a lag model, which means that includes the spatial lag of the dependent variable on the right-hand side of the equation. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional. The default most basic model to be run would be:

```
>>> reg=GM_Lag(y, X, w=w, w_lags=2, name_x=['inc', 'crime'], name_y='hval', name_
↪ds='columbus')
>>> reg.betas
array([[ 45.30170561],
       [  0.62088862],
       [-0.48072345],
       [  0.02836221]])
```

Once the model is run, we can obtain the standard error of the coefficient estimates by calling the diagnostics module:

```
>>> D.se_betas(reg)
array([ 17.91278862,  0.52486082,  0.1822815 ,  0.31740089])
```

But we can also run models that incorporates corrected standard errors following the White procedure. For that, we will have to include the optional parameter `robust='white'`:

```
>>> reg=GM_Lag(y, X, w=w, w_lags=2, robust='white', name_x=['inc', 'crime'], name_
↪y='hval', name_ds='columbus')
>>> reg.betas
array([[ 45.30170561],
       [  0.62088862],
       [-0.48072345],
       [  0.02836221]])
```

And we can access the standard errors from the model object:

```
>>> reg.std_err
array([ 20.47077481,  0.50613931,  0.20138425,  0.38028295])
```

The class is flexible enough to accomodate a spatial lag model that, besides the spatial lag of the dependent variable, includes other non-spatial endogenous regressors. As an example, we will assume that CRIME is actually endogenous and we decide to instrument for it with DISCBD (distance to the CBD). We reload the X including INC only and define CRIME as endogenous and DISCBD as instrument:

```
>>> X = np.array(db.by_col("INC"))
>>> X = np.reshape(X, (49,1))
>>> yd = np.array(db.by_col("CRIME"))
>>> yd = np.reshape(yd, (49,1))
>>> q = np.array(db.by_col("DISCBD"))
>>> q = np.reshape(q, (49,1))
```

And we can run the model again:

```
>>> reg=GM_Lag(y, X, w=w, yend=yd, q=q, w_lags=2, name_x=['inc'], name_y='hoval',
↳name_yend=['crime'], name_q=['discbd'], name_ds='columbus')
>>> reg.betas
array([[ 100.79359082],
       [ -0.50215501],
       [ -1.14881711],
       [ -0.38235022]])
```

Once the model is run, we can obtain the standard error of the coefficient estimates by calling the diagnostics module:

```
>>> D.se_betas(reg)
array([ 53.0829123 ,  1.02511494,  0.57589064,  0.59891744])
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_pred** [array] nx1 array of residuals (using reduced form)

**predy** [array] nx1 array of predicted y values

**predy\_e** [array] nx1 array of predicted y values (using reduced form)

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**kstar** [integer] Number of endogenous variables.

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments

**z** [array] nxk array of variables (combination of x and yend)

**h** [array] nx1 array of instruments (combination of x and q)

**robust** [string] Adjustment for robust standard errors

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form))

**utu** [float] Sum of squared residuals

**sig2** [float] Sigma squared used in computations

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**ak\_test** [tuple] Anselin-Kelejian test; tuple contains the pair (statistic, p-value)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**sig2n** [float] Sigma squared (computed with n in the denominator)

**sig2n\_k** [float] Sigma squared (computed with n-k in the denominator)

**hth** [float]  $H'H$

**hthi** [float]  $(H'H)^{-1}$

**varb** [array]  $(Z'H(H'H)^{-1}H'Z)^{-1}$

**zthhthi** [array]  $Z'H(H'H)^{-1}$

**pfora1a2** [array]  $n(zthhthi)'varb$

**\_\_init\_\_** (self, y, x, yend=None, q=None, w=None, w\_lags=1, lag\_q=True, robust=None, gwkw=None, sig2n\_k=False, spat\_diag=False, vm=False, name\_y=None, name\_x=None, name\_yend=None, name\_q=None, name\_w=None, name\_gwk=None, name\_ds=None)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|   |                  |
|---|------------------|
| <b>__init__</b> (self, y, x[, yend, q, w, w_lags, ...]) | Initialize self. |
|---|------------------|

---

## Attributes

|          |
|----------|
| mean_y   |
| pforala2 |
| sig2n    |
| sig2n_k  |
| std_y    |
| utu      |
| vm       |

### 3.5 spreg.GM\_Error

**class** `spreg.GM_Error`(*y*, *x*, *w*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_w=None*, *name\_ds=None*)  
 GMM method for a spatial error model, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- w** [pysal W object] Spatial weights object (always needed)
- vm** [boolean] If True, include variance-covariance matrix in summary results
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output

#### Examples

We first need to import the needed modules, namely `numpy` to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import libpysal
>>> import numpy as np
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> dbf = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be an numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array([dbf.by_col('HOVAL')]).T
```

Extract CRIME (crime) and INC (income) vectors from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent

variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> names_to_extract = ['INC', 'CRIME']
>>> x = np.array([dbf.by_col(name) for name in names_to_extract]).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will use `columbus.gal`, which contains contiguity relationships between the observations in the Columbus dataset we are using throughout this example. Note that, in order to read the file, not only to open it, we need to append `‘.read()’` at the end of the command.

```
>>> w = libpysal.io.open(libpysal.examples.get_path("columbus.gal"), 'r').read()
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform='r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> model = GM_Error(y, x, w=w, name_y='hoval', name_x=['income', 'crime'], name_
↳ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. Note that because we are running the classical GMM error model from 1998/99, the spatial parameter is obtained as a point estimate, so although you get a value for it (there are for coefficients under `model.betas`), you cannot perform inference on it (there are only three values in `model.se_betas`).

```
>>> print model.name_x
['CONSTANT', 'income', 'crime', 'lambda']
>>> np.around(model.betas, decimals=4)
array([[ 47.6946],
       [  0.7105],
       [-0.5505],
       [  0.3257]])
>>> np.around(model.std_err, decimals=4)
array([ 12.412 ,  0.5044,  0.1785])
>>> np.around(model.z_stat, decimals=6)
array([[ 3.84261100e+00,  1.22000000e-04],
       [ 1.40839200e+00,  1.59015000e-01],
       [-3.08424700e+00,  2.04100000e-03]])
>>> round(model.sig2, 4)
198.5596
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**vm** [array] Variance covariance matrix (kxk)

**sig2** [float] Sigma squared used in computations

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**\_\_init\_\_** (*self*, *y*, *x*, *w*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_w=None*, *name\_ds=None*)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

|   |                  |
|---|------------------|
| <code>__init__(self, y, x, w[, vm, name_y, ...])</code> | Initialize self. |
|---|------------------|

## Attributes

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

## 3.6 spreg.GM\_Error\_Het

**class** `spreg.GM_Error_Het` (*y*, *x*, *w*, *max\_iter=1*, *epsilon=1e-05*, *step1c=False*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_w=None*, *name\_ds=None*)  
 GMM method for a spatial error model with heteroskedasticity, with results and diagnostics; based on [ADKP10], following [Ans11].

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**w** [pysal W object] Spatial weights object

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**step1c** [boolean] If True, then include Step 1c from [ADKP10].

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using libpysal.io.open(). This is the DBF associated with the Columbus shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) and CRIME (crime) vectors from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("CRIME"))
>>> X = np.array(X).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.



```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↳shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Error_Het(y, X, w=w, step1c=True, name_y='home value', name_x=[
↳'income', 'crime'], name_ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that explicitly accounts for heteroskedasticity and that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter.

```
>>> print reg.name_x
['CONSTANT', 'income', 'crime', 'lambda']
```

Hence, we find the same number of betas as of standard errors, which we calculate taking the square root of the diagonal of the variance-covariance matrix:

```
>>> print np.around(np.hstack((reg.betas, np.sqrt(reg.vm.diagonal()).reshape(4,
↳1))), 4)
[[ 47.9963  11.479 ]
 [  0.7105  0.3681]
 [ -0.5588  0.1616]
 [  0.4118  0.168 ]]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10].

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**vm** [array] Variance covariance matrix (kxk)

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**xtx** [float]  $X'X$

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**\_\_init\_\_** (*self*, *y*, *x*, *w*, *max\_iter*=1, *epsilon*=1e-05, *step1c*=False, *vm*=False, *name\_y*=None, *name\_x*=None, *name\_w*=None, *name\_ds*=None)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <b>__init__</b> ( <i>self</i> , <i>y</i> , <i>x</i> , <i>w</i> [, <i>max_iter</i> , <i>epsilon</i> , ...]) | Initialize self. |
|--|------------------|

---

## Attributes

---

|               |
|---------------|
| <b>mean_y</b> |
| <b>std_y</b>  |

---

## 3.7 spreg.GM\_Error\_Hom

**class** `spreg.GM_Error_Hom` (*y*, *x*, *w*, *max\_iter*=1, *epsilon*=1e-05, *A1*='hom\_sc', *vm*=False, *name\_y*=None, *name\_x*=None, *name\_w*=None, *name\_ds*=None)  
 GMM method for a spatial error model with homoskedasticity, with results and diagnostics; based on Drukker et al. (2013) [DEP13], following Anselin (2011) [Ans11].

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**w** [pysal W object] Spatial weights object

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**A1** [string] If A1='het', then the matrix A1 is defined as in Arraiz et al. If A1='hom', then as in [Ans11]. If A1='hom\_sc' (default), then as in [DEP13] and [DPR13].

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using libpysal.io.open(). This is the DBF associated with the Columbus shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) and CRIME (crime) vectors from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("CRIME"))
>>> X = np.array(X).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from columbus.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminars, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Error_Hom(y, X, w=w, A1='hom_sc', name_y='home value', name_x=[
↳ 'income', 'crime'], name_ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that assumes homoskedasticity but that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. This is why you obtain as many coefficient estimates as standard errors, which you calculate taking the square root of the diagonal of the variance-covariance matrix of the parameters:

```
>>> print np.around(np.hstack((reg.betas, np.sqrt(reg.vm.diagonal()).reshape(4,
↳ 1))), 4)
[[ 47.9479  12.3021]
 [  0.7063   0.4967]
 [ -0.556   0.179 ]
 [  0.4129   0.1835]]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array]  $k \times 1$  array of estimated coefficients

**u** [array]  $n \times 1$  array of residuals

**e\_filtered** [array]  $n \times 1$  array of spatially filtered residuals

**predy** [array]  $n \times 1$  array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array]  $n \times 1$  array for dependent variable

**x** [array] Two dimensional array with  $n$  rows and one column for each independent (exogenous) variable, including the constant

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].

**iteration** [integer] Number of iterations of steps 2a and 2b from Arraiz et al.

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**vm** [array] Variance covariance matrix ( $k \times k$ )

**sig2** [float] Sigma squared used in computations

**std\_err** [array]  $1 \times k$  array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**xtx** [float]  $X'X$

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

```
__init__(self, y, x, w, max_iter=1, epsilon=1e-05, A1='hom_sc', vm=False, name_y=None,
          name_x=None, name_w=None, name_ds=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|   |                  |
|---|------------------|
| <code><b>__init__</b>(self, y, x, w[, max_iter, epsilon, ...])</code> | Initialize self. |
|---|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
|---------------------|

---

|                    |
|--------------------|
| <code>std_y</code> |
|--------------------|

---

## 3.8 spreg.GM\_Combo

```
class spreg.GM_Combo(y, x, yend=None, q=None, w=None, w_lags=1, lag_q=True, vm=False,
                     name_y=None, name_x=None, name_yend=None, name_q=None,
                     name_w=None, name_ds=None)
```

GMM method for a spatial lag and error model with endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**w** [pysal W object] Spatial weights object (always needed)

**w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.

**lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely `numpy` to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("columbus.dbf"), 'r')
```

Extract the CRIME column (crime rates) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

The `Combo` class runs an SARAR model, that is a spatial lag+error model. In this case we will run a simple version of that, where we have the spatial effects as well as exogenous variables. Since it is a spatial model, we have to pass in the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Combo(y, X, w=w, name_y='crime', name_x=['income'], name_ds='columbus
↪')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. Note that because we are running the classical GMM error

model from 1998/99, the spatial parameter is obtained as a point estimate, so although you get a value for it (there are for coefficients under `model.betas`), you cannot perform inference on it (there are only three values in `model.se_betas`). Also, this regression uses a two stage least squares estimation method that accounts for the endogeneity created by the spatial lag of the dependent variable. We can check the betas:

```
>>> print reg.name_z
['CONSTANT', 'income', 'W_crime', 'lambda']
>>> print np.around(np.hstack((reg.betas[:-1], np.sqrt(reg.vm.diagonal()))
↳ reshape(3,1))), 3)
[[ 39.059  11.86 ]
 [ -1.404   0.391]
 [  0.467   0.2  ]]
```

And lambda:

```
>>> print 'lambda: ', np.around(reg.betas[-1], 3)
lambda:  [-0.048]
```

This class also allows the user to run a spatial lag+error model with the extra feature of including non-spatial endogenous regressors. This means that, in addition to the spatial lag and error, we consider some of the variables on the right-hand side of the equation as endogenous and we instrument for this. As an example, we will include HOVAL (home value) as endogenous and will instrument with DISCBD (distance to the CSB). We first need to read in the variables:

```
>>> yd = []
>>> yd.append(db.by_col("HOVAL"))
>>> yd = np.array(yd).T
>>> q = []
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

And then we can run and explore the model analogously to the previous combo:

```
>>> reg = GM_Combo(y, X, yd, q, w=w, name_x=['inc'], name_y='crime', name_yend=[
↳ 'hval'], name_q=['discbd'], name_ds='columbus')
>>> print reg.name_z
['CONSTANT', 'inc', 'hval', 'W_crime', 'lambda']
>>> names = np.array(reg.name_z).reshape(5,1)
>>> print np.hstack((names[0:4,:], np.around(np.hstack((reg.betas[:-1], np.
↳ sqrt(reg.vm.diagonal()).reshape(4,1))), 4)))
[['CONSTANT' '50.0944' '14.3593']
 ['inc' '-0.2552' '0.5667']
 ['hval' '-0.6885' '0.3029']
 ['W_crime' '0.4375' '0.2314']]
```

```
>>> print 'lambda: ', np.around(reg.betas[-1], 3)
lambda:  [ 0.254]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**e\_pred** [array] nx1 array of residuals (using reduced form)

**predy** [array] nx1 array of predicted y values

**predy\_e** [array] nx1 array of predicted y values (using reduced form)

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**z** [array] nxk array of variables (combination of x and yend)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form))

**sig2** [float] Sigma squared used in computations (based on filtered residuals)

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in ouput

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**\_\_init\_\_** (*self*, y, x, yend=None, q=None, w=None, w\_lags=1, lag\_q=True, vm=False, name\_y=None, name\_x=None, name\_yend=None, name\_q=None, name\_w=None, name\_ds=None)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x[, yend, q, w, w_lags, ...])</code> | Initialize self. |
|--|------------------|

---



## Attributes

---

|        |
|--------|
| mean_y |
| std_y  |

---

## 3.9 spreg.GM\_Combo\_Het

```
class spreg.GM_Combo_Het (y, x, yend=None, q=None, w=None, w_lags=1, lag_q=True, max_iter=1,
                           epsilon=1e-05, step1c=False, inv_method='power_exp', vm=False,
                           name_y=None, name_x=None, name_yend=None, name_q=None,
                           name_w=None, name_ds=None)
```

GMM method for a spatial lag and error model with heteroskedasticity and endogenous variables, with results and diagnostics; based on [ADKP10], following [Ans11].

### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)
- w** [pysal W object] Spatial weights object (always needed)
- w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.
- lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).
- max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.
- epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.
- step1c** [boolean] If True, then include Step 1c from [ADKP10].
- inv\_method** [string] If “power\_exp”, then compute inverse using the power expansion. If “true\_inv”, then compute the true inverse. Note that true\_inv will fail for large n.
- vm** [boolean] If True, include variance-covariance matrix in summary results
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_yend** [list of strings] Names of endogenous variables for use in output
- name\_q** [list of strings] Names of instruments for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely `numpy` to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be an numpy array of shape `(n, 1)` as opposed to the also common shape of `(n, )` that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an `nxj` numpy array, where `j` is the number of independent variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing `gal` file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

The `Combo` class runs an SARAR model, that is a spatial lag+error model. In this case we will run a simple version of that, where we have the spatial effects as well as exogenous variables. Since it is a spatial model, we have to pass in the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Combo_Het(y, X, w=w, step1c=True, name_y='hoval', name_x=['income'],
↪name_ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that explicitly accounts for heteroskedasticity and that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. Hence, we find the same number of betas as of standard errors, which we calculate taking the square root of the diagonal of the variance-covariance matrix:

```
>>> print reg.name_z
['CONSTANT', 'income', 'W_hoval', 'lambda']
>>> print np.around(np.hstack((reg.betas, np.sqrt(reg.vm.diagonal()).reshape(4,
↪1))), 4)
[[ 9.9753 14.1435]
 [ 1.5742  0.374 ]
 [ 0.1535  0.3978]
 [ 0.2103  0.3924]]
```

This class also allows the user to run a spatial lag+error model with the extra feature of including non-spatial endogenous regressors. This means that, in addition to the spatial lag and error, we consider some of the variables on the right-hand side of the equation as endogenous and we instrument for this. As an example, we will include CRIME (crime rates) as endogenous and will instrument with DISCBD (distance to the CSB). We first need to read in the variables:

```
>>> yd = []
>>> yd.append(db.by_col("CRIME"))
>>> yd = np.array(yd).T
>>> q = []
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

And then we can run and explore the model analogously to the previous combo:

```
>>> reg = GM_Combo_Het(y, X, yd, q, w=w, steplc=True, name_x=['inc'], name_y=
↪'hovel', name_yend=['crime'], name_q=['discbd'], name_ds='columbus')
>>> print reg.name_z
['CONSTANT', 'inc', 'crime', 'W_hoval', 'lambda']
>>> print np.round(reg.betas, 4)
[[ 113.9129]
 [ -0.3482]
 [ -1.3566]
 [ -0.5766]
 [  0.6561]]
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- e\_pred** [array] nx1 array of residuals (using reduced form)
- predy** [array] nx1 array of predicted y values
- predy\_e** [array] nx1 array of predicted y values (using reduced form)
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments

**z** [array] nxk array of variables (combination of x and yend)

**h** [array] nxl array of instruments (combination of x and q)

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10].

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form))

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in ouput

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**hth** [float]  $H'H$

**\_\_init\_\_** (*self*, y, x, yend=None, q=None, w=None, w\_lags=1, lag\_q=True, max\_iter=1, epsilon=1e-05, step1c=False, inv\_method='power\_exp', vm=False, name\_y=None, name\_x=None, name\_yend=None, name\_q=None, name\_w=None, name\_ds=None)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x[, yend, q, w, w_lags, ...])</code> | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

## 3.10 spreg.GM\_Combo\_Hom

```
class spreg.GM_Combo_Hom(y, x, yend=None, q=None, w=None, w_lags=1, lag_q=True, max_iter=1,
                        epsilon=1e-05, A1='hom_sc', vm=False, name_y=None, name_x=None,
                        name_yend=None, name_q=None, name_w=None, name_ds=None)
```

GMM method for a spatial lag and error model with homoskedasticity and endogenous variables, with results and diagnostics; based on Drukker et al. (2013) [DEP13], following Anselin (2011) [Ans11].

### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)
- w** [pysal W object] Spatial weights object (always necessary)
- w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.
- lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).
- max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.
- epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.
- A1** [string] If A1='het', then the matrix A1 is defined as in [ADKP10]. If A1='hom', then as in [Ans11]. If A1='hom\_sc' (default), then as in [DEP13] and [DPR13].
- vm** [boolean] If True, include variance-covariance matrix in summary results
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_yend** [list of strings] Names of endogenous variables for use in output
- name\_q** [list of strings] Names of instruments for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output

### Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using libpysal.io.open(). This is the DBF associated with the Columbus shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

Example only with spatial lag

The Combo class runs an SARAR model, that is a spatial lag+error model. In this case we will run a simple version of that, where we have the spatial effects as well as exogenous variables. Since it is a spatial model, we have to pass in the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Combo_Hom(y, X, w=w, A1='hom_sc', name_x=['inc'], name_y=
↪'hoval', name_yend=['crime'], name_q=['discbd'], name_ds='columbus')
>>> print np.around(np.hstack((reg.betas, np.sqrt(reg.vcov.diagonal()).reshape(4,
↪1))), 4)
[[ 10.1254  15.2871]
 [  1.5683   0.4407]
 [  0.1513   0.4048]
 [  0.2103   0.4226]]
```

This class also allows the user to run a spatial lag+error model with the extra feature of including non-spatial endogenous regressors. This means that, in addition to the spatial lag and error, we consider some of the variables on the right-hand side of the equation as endogenous and we instrument for this. As an example, we will include CRIME (crime rates) as endogenous and will instrument with DISCBD (distance to the CSB). We first need to read in the variables:

```
>>> yd = []
>>> yd.append(db.by_col("CRIME"))
>>> yd = np.array(yd).T
>>> q = []
```

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```
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

And then we can run and explore the model analogously to the previous combo:

```
>>> reg = GM_Combo_Hom(y, X, yd, q, w=w, A1='hom_sc', name_ds=
↳ 'columbus')
>>> betas = np.array(['CONSTANT'], ['inc'], ['crime'], ['W_hoval'], ['lambda']))
>>> print np.hstack((betas, np.around(np.hstack((reg.betas, np.sqrt(reg.vm.
↳ diagonal()).reshape(5,1))), 5)))
[['CONSTANT' '111.7705' '67.75191']
 ['inc' '-0.30974' '1.16656']
 ['crime' '-1.36043' '0.6841']
 ['W_hoval' '-0.52908' '0.84428']
 ['lambda' '0.60116' '0.18605']]
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- e\_pred** [array] nx1 array of residuals (using reduced form)
- predy** [array] nx1 array of predicted y values
- predy\_e** [array] nx1 array of predicted y values (using reduced form)
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments
- z** [array] nxk array of variables (combination of x and yend)
- h** [array] nx1 array of instruments (combination of x and q)
- iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [\[ADKP10\]](#).
- iteration** [integer] Number of iterations of steps 2a and 2b from [\[ADKP10\]](#).
- mean\_y** [float] Mean of dependent variable
- std\_y** [float] Standard deviation of dependent variable
- vm** [array] Variance covariance matrix (kxk)
- pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form))

**sig2** [float] Sigma squared used in computations (based on filtered residuals)

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**hth** [float]  $H'H$

```
__init__(self, y, x, yend=None, q=None, w=None, w_lags=1, lag_q=True, max_iter=1, epsilon=1e-05, A1='hom_sc', vm=False, name_y=None, name_x=None, name_yend=None, name_q=None, name_w=None, name_ds=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x[, yend, q, w, w_lags, ...])</code> | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

## 3.11 spreg.GM\_Endog\_Error

```
class spreg.GM_Endog_Error(y, x, yend, q, w, vm=False, name_y=None, name_x=None,
                           name_yend=None, name_q=None, name_w=None, name_ds=None)
GMM method for a spatial error model with endogenous variables, with results and diagnostics; based on
Kelejian and Prucha (1998, 1999) [KP98] [KP99].
```

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable



**q** [array] Two dimensional array with *n* rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from *x*)

**w** [pysal W object] Spatial weights object (always needed)

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import libpysal
>>> import numpy as np
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> dbf = libpysal.io.open(libpysal.examples.get_path("columbus.dbf"), 'r')
```

Extract the CRIME column (crime rates) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (*n*, 1) as opposed to the also common shape of (*n*,) that other packages accept.

```
>>> y = np.array([dbf.by_col('CRIME')]).T
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an *n*×*j* numpy array, where *j* is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x = np.array([dbf.by_col('INC')]).T
```

In this case we consider HOVAL (home value) is an endogenous regressor. We tell the model that this is so by passing it in a different parameter from the exogenous variables (*x*).

```
>>> yend = np.array([dbf.by_col('HOVAL')]).T
```

Because we have endogenous variables, to obtain a correct estimate of the model, we need to instrument for HOVAL. We use DISCBD (distance to the CBD) for this and hence put it in the instruments parameter, 'q'.

```
>>> q = np.array([dbf.by_col('DISCBD')]).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing `gal` file or create a new one. In this case, we will use `columbus.gal`, which contains contiguity

relationships between the observations in the Columbus dataset we are using throughout this example. Note that, in order to read the file, not only to open it, we need to append `‘.read()’` at the end of the command.

```
>>> w = libpysal.io.open(libpysal.examples.get_path("columbus.gal"), 'r').read()
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform='r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous), the instruments and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> from spreg import GM_Endog_Error
>>> model = GM_Endog_Error(y, x, yend, q, w=w, name_x=['inc'], name_y='crime',
↪ name_yend=['hoval'], name_q=['discbd'], name_ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. Note that because we are running the classical GMM error model from 1998/99, the spatial parameter is obtained as a point estimate, so although you get a value for it (there are for coefficients under `model.betas`), you cannot perform inference on it (there are only three values in `model.se_betas`). Also, this regression uses a two stage least squares estimation method that accounts for the endogeneity created by the endogenous variables included.

```
>>> print model.name_z
['CONSTANT', 'inc', 'hoval', 'lambda']
>>> np.around(model.betas, decimals=4)
array([[ 82.573 ],
       [  0.581 ],
       [-1.4481],
       [  0.3499]])
>>> np.around(model.std_err, decimals=4)
array([ 16.1381,  1.3545,  0.7862])
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**z** [array] nxk array of variables (combination of x and yend)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**sig2** [float] Sigma squared used in computations

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**\_\_init\_\_** (*self*, *y*, *x*, *yend*, *q*, *w*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_yend=None*, *name\_q=None*, *name\_w=None*, *name\_ds=None*)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__</code> ( <i>self</i> , <i>y</i> , <i>x</i> , <i>yend</i> , <i>q</i> , <i>w</i> , <i>vm</i> , ...) | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

## 3.12 spreg.GM\_Endog\_Error\_Het

**class** `spreg.GM_Endog_Error_Het` (*y*, *x*, *yend*, *q*, *w*, *max\_iter=1*, *epsilon=1e-05*, *step1c=False*, *inv\_method='power\_exp'*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_yend=None*, *name\_q=None*, *name\_w=None*, *name\_ds=None*)

GMM method for a spatial error model with heteroskedasticity and endogenous variables, with results and diagnostics; based on [ADKP10], following [Ans11].

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**w** [pysal W object] Spatial weights object

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**step1c** [boolean] If True, then include Step 1c from [ADKP10].

**inv\_method** [string] If “power\_exp”, then compute inverse using the power expansion. If “true\_inv”, then compute the true inverse. Note that true\_inv will fail for large n.

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using libpysal.io.open(). This is the DBF associated with the Columbus shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

In this case we consider CRIME (crime rates) is an endogenous regressor. We tell the model that this is so by passing it in a different parameter from the exogenous variables (x).

```
>>> yd = []
>>> yd.append(db.by_col("CRIME"))
>>> yd = np.array(yd).T
```

Because we have endogenous variables, to obtain a correct estimate of the model, we need to instrument for CRIME. We use DISCBD (distance to the CBD) for this and hence put it in the instruments parameter, 'q'.

```
>>> q = []
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous), the instruments and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Endog_Error_Het(y, X, yd, q, w=w, step1c=True, name_x=['inc'], name_
↪y='hval', name_yend=['crime'], name_q=['discbd'], name_ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that explicitly accounts for heteroskedasticity and that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. Hence, we find the same number of betas as of standard errors, which we calculate taking the square root of the diagonal of the variance-covariance matrix:

```
>>> print reg.name_z
['CONSTANT', 'inc', 'crime', 'lambda']
>>> print np.around(np.hstack((reg.betas, np.sqrt(reg.vm.diagonal()).reshape(4,
↪1))), 4)
[[ 55.3971  28.8901]
 [  0.4656  0.7731]
 [ -0.6704  0.468 ]
 [  0.4114  0.1777]]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array]  $k \times 1$  array of estimated coefficients

**u** [array]  $n \times 1$  array of residuals

**e\_filtered** [array]  $n \times 1$  array of spatially filtered residuals

**predy** [array]  $n \times 1$  array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array]  $n \times 1$  array for dependent variable

**x** [array] Two dimensional array with  $n$  rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with  $n$  rows and one column for each endogenous variable

**q** [array] Two dimensional array with  $n$  rows and one column for each external exogenous variable used as instruments

**z** [array]  $n \times k$  array of variables (combination of  $x$  and  $yend$ )

**h** [array]  $n \times l$  array of instruments (combination of  $x$  and  $q$ )

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10].

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix ( $k \times k$ )

**pr2** [float] Pseudo R squared (squared correlation between  $y$  and  $ypred$ )

**std\_err** [array]  $1 \times k$  array of standard errors of the betas

**z\_stat** [list of tuples]  $z$  statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**hth** [float]  $H'H$

**\_\_init\_\_** (*self*, *y*, *x*, *yend*, *q*, *w*, *max\_iter*=1, *epsilon*=1e-05, *step1c*=False, *inv\_method*='power\_exp', *vm*=False, *name\_y*=None, *name\_x*=None, *name\_yend*=None, *name\_q*=None, *name\_w*=None, *name\_ds*=None)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x, yend, q, w[, max_iter, ...])</code> | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

## 3.13 spreg.GM\_Endog\_Error\_Hom

```
class spreg.GM_Endog_Error_Hom(y, x, yend, q, w, max_iter=1, epsilon=1e-05, A1='hom_sc',
                               vm=False, name_y=None, name_x=None, name_yend=None,
                               name_q=None, name_w=None, name_ds=None)
```

GMM method for a spatial error model with homoskedasticity and endogenous variables, with results and diagnostics; based on Drukker et al. (2013) [DEP13], following Anselin (2011) [Ans11].

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**w** [pysal W object] Spatial weights object

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**A1** [string] If A1='het', then the matrix A1 is defined as in [ADKP10]. If A1='hom', then as in [Ans11]. If A1='hom\_sc' (default), then as in [DEP13] and [DPR13].

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

### Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Extract the HOVAL column (home values) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be an numpy array of shape  $(n, 1)$  as opposed to the also common shape of  $(n, )$  that other packages accept.

```
>>> y = np.array(db.by_col("HOVAL"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an  $n \times j$  numpy array, where  $j$  is the number of independent variables (not including a constant). By default this class adds a vector of ones to the independent variables passed in.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

In this case we consider CRIME (crime rates) is an endogenous regressor. We tell the model that this is so by passing it in a different parameter from the exogenous variables ( $x$ ).

```
>>> yd = []
>>> yd.append(db.by_col("CRIME"))
>>> yd = np.array(yd).T
```

Because we have endogenous variables, to obtain a correct estimate of the model, we need to instrument for CRIME. We use DISCBD (distance to the CBD) for this and hence put it in the instruments parameter, 'q'.

```
>>> q = []
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous), the instruments and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.



```
>>> reg = GM_Endog_Error_Hom(y, X, yd, q, w=w, A1='hom_sc', name_x=['inc'], name_
↪ y='hoval', name_yend=['crime'], name_q=['discbd'], name_ds='columbus')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that assumes homoskedasticity but that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. Hence, we find the same number of betas as of standard errors, which we calculate taking the square root of the diagonal of the variance-covariance matrix:

```
>>> print reg.name_z
['CONSTANT', 'inc', 'crime', 'lambda']
>>> print np.around(np.hstack((reg.betas, np.sqrt(reg.vm.diagonal()).reshape(4,
↪ 1))), 4)
[[ 55.3658  23.496 ]
 [  0.4643  0.7382]
 [-0.669   0.3943]
 [  0.4321  0.1927]]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments

**z** [array] nxk array of variables (combination of x and yend)

**h** [array] nx1 array of instruments (combination of x and q)

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10].

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**sig2** [float] Sigma squared used in computations

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**hth** [float]  $H'H$

**\_\_init\_\_** (*self*, *y*, *x*, *yend*, *q*, *w*, *max\_iter*=1, *epsilon*=1e-05, *A1*='hom\_sc', *vm*=False, *name\_y*=None, *name\_x*=None, *name\_yend*=None, *name\_q*=None, *name\_w*=None, *name\_ds*=None)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

**\_\_init\_\_**(*self*, *y*, *x*, *yend*, *q*, *w*[, *max\_iter*, ...]) Initialize self.

---

## Attributes

---

**mean\_y**

---

**std\_y**

---

## 3.14 spreg.TSLS

**class** **spreg.TSLS** (*y*, *x*, *yend*, *q*, *w*=None, *robust*=None, *gwk*=None, *sig2n\_k*=False, *spat\_diag*=False, *vm*=False, *name\_y*=None, *name\_x*=None, *name\_yend*=None, *name\_q*=None, *name\_w*=None, *name\_gwk*=None, *name\_ds*=None)

Two stage least squares with results and diagnostics.

### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**w** [pysal W object] Spatial weights object (required if running spatial diagnostics)

**robust** [string] If 'white', then a White consistent estimator of the variance-covariance matrix is given. If 'hac', then a HAC consistent estimator of the variance-covariance matrix is given.

Default set to None.

**gwk** [pysal W object] Kernel spatial weights needed for HAC estimation. Note: matrix must have ones along the main diagonal.

**sig2n\_k** [boolean] If True, then use  $n-k$  to estimate  $\sigma^2$ . If False, use  $n$ .

**spat\_diag** [boolean] If True, then compute Anselin-Kelejian test (requires *w*)

**vm** [boolean] If True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

## Examples

We first need to import the needed modules, namely `numpy` to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on Columbus neighborhood crime (49 areas) using `libpysal.io.open()`. This is the DBF associated with the Columbus shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("columbus.dbf"), 'r')
```

Extract the CRIME column (crime rates) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape  $(n, 1)$  as opposed to the also common shape of  $(n, )$  that other packages accept.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an  $n \times j$  numpy array, where  $j$  is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in, but this can be overridden by passing `constant=False`.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

In this case we consider HOVAL (home value) is an endogenous regressor. We tell the model that this is so by passing it in a different parameter from the exogenous variables (*x*).

```
>>> yd = []
>>> yd.append(db.by_col("HOVAL"))
>>> yd = np.array(yd).T
```

Because we have endogenous variables, to obtain a correct estimate of the model, we need to instrument for HOVAL. We use DISCBD (distance to the CBD) for this and hence put it in the instruments parameter, 'q'.

```
>>> q = []
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous) and the instruments. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = TSLS(y, X, yd, q, name_x=['inc'], name_y='crime', name_yend=['hoval'],
↳name_q=['discbd'], name_ds='columbus')
>>> print reg.betas
[[ 88.46579584]
 [  0.5200379 ]
 [-1.58216593]]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)

**kstar** [integer] Number of endogenous variables.

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments

**z** [array] nxk array of variables (combination of x and yend)

**h** [array] nx1 array of instruments (combination of x and q)

**robust** [string] Adjustment for robust standard errors

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred)

**utu** [float] Sum of squared residuals

**sig2** [float] Sigma squared used in computations

**std\_err** [array] 1xk array of standard errors of the betas

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float

**ak\_test** [tuple] Anselin-Kelejian test; tuple contains the pair (statistic, p-value)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in ouput

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**title** [string] Name of the regression method used

**sig2n** [float] Sigma squared (computed with n in the denominator)

**sig2n\_k** [float] Sigma squared (computed with n-k in the denominator)

**hth** [float]  $H'H$

**hthi** [float]  $(H'H)^{-1}$

**varb** [array]  $(Z'H(H'H)^{-1}H'Z)^{-1}$

**zthhthi** [array]  $Z'H(H'H)^{-1}$

**pfora1a2** [array]  $n(zthhthi)'varb$

**\_\_init\_\_** (*self*, *y*, *x*, *yend*, *q*, *w=None*, *robust=None*, *gwk=None*, *sig2n\_k=False*, *spat\_diag=False*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_yend=None*, *name\_q=None*, *name\_w=None*, *name\_gwk=None*, *name\_ds=None*)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x, yend, q[, w, robust, ...])</code> | Initialize self. |
|--|------------------|

---

## Attributes

---

|                       |
|-----------------------|
| <code>mean_y</code>   |
| <code>pfora1a2</code> |
| <code>sig2n</code>    |
| <code>sig2n_k</code>  |
| <code>std_y</code>    |
| <code>utu</code>      |

---

Continued on next page

Table 29 – continued from previous page

vm

## 3.15 spreg.ThreeSLS

```
class spreg.ThreeSLS(bigy, bigX, bigyend, bigq, regimes=None, nonspat_diag=True,  
                    name_bigy=None, name_bigX=None, name_bigyend=None,  
                    name_bigq=None, name_ds=None, name_regimes=None)
```

User class for 3SLS estimation

### Parameters

- bigy** [dictionary] with vector for dependent variable by equation
- bigX** [dictionary] with matrix of explanatory variables by equation (note, already includes constant term)
- bigyend** [dictionary] with matrix of endogenous variables by equation
- bigq** [dictionary] with matrix of instruments by equation
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- nonspat\_diag: boolean** flag for non-spatial diagnostics, default = True.
- name\_bigy** [dictionary] with name of dependent variable for each equation. default = None, but should be specified. is done when sur\_stackxy is used
- name\_bigX** [dictionary] with names of explanatory variables for each equation. default = None, but should be specified. is done when sur\_stackxy is used
- name\_bigyend** [dictionary] with names of endogenous variables for each equation. default = None, but should be specified. is done when sur\_stackZ is used
- name\_bigq** [dictionary] with names of instrumental variables for each equation. default = None, but should be specified. is done when sur\_stackZ is used.
- name\_ds** [string] name for the data set.
- name\_regimes** [string] name of regime variable for use in the output.

### Examples

First import libpysal to load the spatial analysis tools.

```
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

The specification of the model to be estimated can be provided as lists. Each equation should be listed separately. In this example, equation 1 has HR80 as dependent variable, PS80 and UE80 as exogenous regressors, RD80 as endogenous regressor and FP79 as additional instrument. For equation 2, HR90 is the dependent variable, PS90 and UE90 the exogenous regressors, RD90 as endogenous regressor and FP99 as additional instrument

```
>>> y_var = ['HR80', 'HR90']
>>> x_var = [['PS80', 'UE80'], ['PS90', 'UE90']]
>>> yend_var = [['RD80'], ['RD90']]
>>> q_var = [['FP79'], ['FP89']]
```

The SUR method requires data to be provided as dictionaries. PySAL provides two tools to create these dictionaries from the list of variables: `sur_dictxy` and `sur_dictZ`. The tool `sur_dictxy` can be used to create the dictionaries for Y and X, and `sur_dictZ` for endogenous variables (yend) and additional instruments (q).

```
>>> bigy, bigX, bigyvars, bigXvars = pysal.spreg.sur_utils.sur_dictxy(db, y_var, x_var)
>>> bigyend, bigyendvars = pysal.spreg.sur_utils.sur_dictZ(db, yend_var)
>>> bigq, bigqvars = pysal.spreg.sur_utils.sur_dictZ(db, q_var)
```

We can now run the regression and then have a summary of the output by typing: `print(reg.summary)`

Alternatively, we can just check the betas and standard errors, asymptotic t and p-value of the parameters:

```
>>> reg = ThreeSLS(bigy, bigX, bigyend, bigq, name_bigy=bigyvars, name_bigX=bigXvars,
↳ name_bigyend=bigyendvars, name_bigq=bigqvars, name_ds="NAT")
>>> reg.b3SLS
{0: array([[ 6.92426353],
           [ 1.42921826],
           [ 0.00049435],
           [ 3.5829275 ]]), 1: array([[ 7.62385875],
           [ 1.65031181],
           [-0.21682974],
           [ 3.91250428]])}
```

```
>>> reg.tsls_inf
{0: array([[ 0.23220853, 29.81916157, 0.          ],
           [ 0.10373417, 13.77770036, 0.          ],
           [ 0.03086193, 0.01601807, 0.98721998],
           [ 0.11131999, 32.18584124, 0.          ]]), 1: array([[ 0.28739415, 26.
↳ 52753638, 0.          ],
           [ 0.09597031, 17.19606554, 0.          ],
           [ 0.04089547, -5.30204786, 0.00000011],
           [ 0.13586789, 28.79638723, 0.          ]])}
```

### Attributes

**bigy** [dictionary] with y values

**bigZ** [dictionary] with matrix of exogenous and endogenous variables for each equation

**bigZHZH** [dictionary] with matrix of cross products  $\hat{Z} \hat{r}' \hat{Z}$

**bigZHy** [dictionary] with matrix of cross products  $\hat{Z} \hat{r}' y_{\text{end}}$

**n\_eq** [int] number of equations

**n** [int] number of observations in each cross-section

**bigK** [array] vector with number of explanatory variables (including constant, exogenous and endogenous) for each equation

**b2SLS** [dictionary] with 2SLS regression coefficients for each equation

**tslsE** [array]  $N \times n_{\text{eq}}$  array with OLS residuals for each equation

**b3SLS** [dictionary] with 3SLS regression coefficients for each equation

**varb** [array] variance-covariance matrix

**sig** [array] Sigma matrix of inter-equation error covariances

**bigE** [array] n by n\_eq array of residuals

**corr** [array] inter-equation 3SLS error correlation matrix

**tsls\_inf** [dictionary] with standard error, asymptotic t and p-value, one for each equation

**surchow** [array] list with tuples for Chow test on regression coefficients each tuple contains test value, degrees of freedom, p-value

**name\_ds** [string] name for the data set

**name\_bigy** [dictionary] with name of dependent variable for each equation

**name\_bigX** [dictionary] with names of explanatory variables for each equation

**name\_bigyend** [dictionary] with names of endogenous variables for each equation

**name\_bigq** [dictionary] with names of instrumental variables for each equations

**name\_regimes** [string] name of regime variable for use in the output

**\_\_init\_\_** (*self*, *bigy*, *bigX*, *bigyend*, *bigq*, *regimes=None*, *nonspat\_diag=True*, *name\_bigy=None*, *name\_bigX=None*, *name\_bigyend=None*, *name\_bigq=None*, *name\_ds=None*, *name\_regimes=None*)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|   |                  |
|---|------------------|
| <code>__init__(self, bigy, bigX, bigyend, bigq[, ...])</code> | Initialize self. |
|---|------------------|

---

## 3.16 Regimes Models

Regimes models are variants of spatial regression models which allow for structural instability in parameters. That means that these models allow different coefficient values in distinct subsets of the data.

|   |  |
|---|--|
| <code>spreg.OLS_Regimes(y, x, regimes[, w, ...])</code>       | Ordinary least squares with results and diagnostics.   |
| <code>spreg.ML_Lag_Regimes(y, x, regimes[, w, ...])</code>    | ML estimation of the spatial lag model with regimes (note no consistency checks, diagnostics or constants added) [Ans88].                  |
| <code>spreg.ML_Error_Regimes(y, x, regimes[, w, ...])</code>  | ML estimation of the spatial error model with regimes (note no consistency checks, diagnostics or constants added); Anselin (1988) [Ans88] |
| <code>spreg.GM_Lag_Regimes(y, x, regimes[, yend, ...])</code> | Spatial two stage least squares (S2SLS) with regimes; [Ans88]  |
| <code>spreg.GM_Error_Regimes(y, x, regimes, w[, ...])</code>  | GMM method for a spatial error model with regimes, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].  |
| <code>spreg.GM_Error_Het_Regimes(y, x, regimes, w)</code>     | GMM method for a spatial error model with heteroskedasticity and regimes; based on Arraiz et al [ADKP10], following Anselin [Ans11].       |

Continued on next page



Table 31 – continued from previous page

|  |  |
|--|--|
| <code>spreg.GM_Error_Hom_Regimes(y, x, regimes, w)</code>      | GMM method for a spatial error model with homoskedasticity, with regimes, results and diagnostics; based on Drukker et al.   |
| <code>spreg.GM_Combo_Regimes(y, x, regimes[, ...])</code>      | GMM method for a spatial lag and error model with regimes and endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].                       |
| <code>spreg.GM_Combo_Hom_Regimes(y, x, regimes[, ...])</code>  | GMM method for a spatial lag and error model with homoskedasticity, regimes and endogenous variables, with results and diagnostics; based on Drukker et al.                                      |
| <code>spreg.GM_Combo_Het_Regimes(y, x, regimes[, ...])</code>  | GMM method for a spatial lag and error model with heteroskedasticity, regimes and endogenous variables, with results and diagnostics; based on Arraiz et al [ADKP10], following Anselin [Ans11]. |
| <code>spreg.GM_Endog_Error_Regimes(y, x, yend, q, ...)</code>  | GMM method for a spatial error model with regimes and endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].                               |
| <code>spreg.GM_Endog_Error_Hom_Regimes(y, x, yend, ...)</code> | GMM method for a spatial error model with homoskedasticity, regimes and endogenous variables.  |
| <code>spreg.GM_Endog_Error_Het_Regimes(y, x, yend, ...)</code> | GMM method for a spatial error model with heteroskedasticity, regimes and endogenous variables, with results and diagnostics; based on Arraiz et al [ADKP10], following Anselin [Ans11].         |

### 3.16.1 spreg.OLS\_Regimes

**class** `spreg.OLS_Regimes` (*y, x, regimes, w=None, robust=None, gwk=None, sig2n\_k=True, nonspat\_diag=True, spat\_diag=False, moran=False, white\_test=False, vm=False, constant\_regi='many', cols2regi='all', regime\_err\_sep=True, cores=False, name\_y=None, name\_x=None, name\_regimes=None, name\_w=None, name\_gwk=None, name\_ds=None*)

Ordinary least squares with results and diagnostics.

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- w** [pysal W object] Spatial weights object (required if running spatial diagnostics)
- robust** [string] If 'white', then a White consistent estimator of the variance-covariance matrix is given. If 'hac', then a HAC consistent estimator of the variance-covariance matrix is given. Default set to None.
- gwk** [pysal W object] Kernel spatial weights needed for HAC estimation. Note: matrix must have ones along the main diagonal.
- sig2n\_k** [boolean] If True, then use  $n-k$  to estimate  $\sigma^2$ . If False, use  $n$ .
- nonspat\_diag** [boolean] If True, then compute non-spatial diagnostics on the regression.

**spat\_diag** [boolean] If True, then compute Lagrange multiplier tests (requires w). Note: see moran for further tests.

**moran** [boolean] If True, compute Moran's I on the residuals. Note: requires spat\_diag=True.

**white\_test** [boolean] If True, compute White's specification robust test. (requires non-spat\_diag=True)

**vm** [boolean] If True, include variance-covariance matrix in summary results

**constant\_regi: string, optional** Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes
- 'many': a vector of ones is appended to x and considered different per regime (default)

**cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = db.by_col(y_var)
>>> y = np.array(y).reshape(len(y), 1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of

independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

We can now run the regression and then have a summary of the output by typing: `olsr.summary` Alternatively, we can just check the betas and standard errors of the parameters:

```
>>> olsr = OLS_Regimes(y, x, regimes, nonspat_diag=False, name_y=y_var, name_x=[
↳ 'PS90', 'UE90'], name_regimes=r_var, name_ds='NAT')
>>> olsr.betas
array([[ 0.39642899],
       [ 0.65583299],
       [ 0.48703937],
       [ 5.59835    ],
       [ 1.16210453],
       [ 0.53163886]])
>>> np.sqrt(olsr.vm.diagonal())
array([ 0.24816345,  0.09662678,  0.03628629,  0.46894564,  0.21667395,
        0.05945651])
>>> olsr.cols2regi
'all'
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- predy** [array] nx1 array of predicted y values
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- robust** [string] Adjustment for robust standard errors Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- mean\_y** [float] Mean of dependent variable
- std\_y** [float] Standard deviation of dependent variable
- vm** [array] Variance covariance matrix (kxk)
- r2** [float] R squared Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

- ar2** [float] Adjusted R squared Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- utu** [float] Sum of squared residuals
- sig2** [float] Sigma squared used in computations Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- sig2ML** [float] Sigma squared (maximum likelihood) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- f\_stat** [tuple] Statistic (float), p-value (float) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- logll** [float] Log likelihood Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- aic** [float] Akaike information criterion Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- schwarz** [float] Schwarz information criterion Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- t\_stat** [list of tuples] t statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- mulColli** [float] Multicollinearity condition number Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- jarque\_bera** [dictionary] 'jb': Jarque-Bera statistic (float); 'pvalue': p-value (float); 'df': degrees of freedom (int) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- breusch\_pagan** [dictionary] 'bp': Breusch-Pagan statistic (float); 'pvalue': p-value (float); 'df': degrees of freedom (int) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- koenker\_bassett** [dictionary] 'kb': Koenker-Bassett statistic (float); 'pvalue': p-value (float); 'df': degrees of freedom (int). Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details).
- white** [dictionary] 'wh': White statistic (float); 'pvalue': p-value (float); 'df': degrees of freedom (int). Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- lm\_error** [tuple] Lagrange multiplier test for spatial error model; tuple contains the pair (statistic, p-value), where each is a float. Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- lm\_lag** [tuple] Lagrange multiplier test for spatial lag model; tuple contains the pair (statistic, p-value), where each is a float. Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- rlm\_error** [tuple] Robust lagrange multiplier test for spatial error model; tuple contains the pair (statistic, p-value), where each is a float. Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

- rlm\_lag** [tuple] Robust lagrange multiplier test for spatial lag model; tuple contains the pair (statistic, p-value), where each is a float. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- lm\_sarma** [tuple] Lagrange multiplier test for spatial SARMA model; tuple contains the pair (statistic, p-value), where each is a float. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- moran\_res** [tuple] Moran’s I for the residuals; tuple containing the triple (Moran’s I, standardized Moran’s I, p-value)
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_gwk** [string] Name of kernel weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output
- name\_regimes** [string] Name of regime variable for use in the output
- title** [string] Name of the regression method used. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- sig2n** [float] Sigma squared (computed with n in the denominator)
- sig2n\_k** [float] Sigma squared (computed with n-k in the denominator)
- xtx** [float]  $X'X$ . Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- xtxi** [float]  $(X'X)^{-1}$ . Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with ‘x’.
- constant\_regi** [string] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:
- ‘one’: a vector of ones is appended to x and held constant across regimes.
  - ‘many’: a vector of ones is appended to x and considered different per regime.
- cols2regi** [list] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If ‘all’, all the variables vary by regime.
- regime\_err\_sep: boolean** If True, a separate regression is run for each regime.
- kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)
- kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate.
- nr** [int] Number of different regimes in the ‘regimes’ list.
- multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression.

```
__init__(self, y, x, regimes, w=None, robust=None, gwk=None, sig2n_k=True, nonspat_diag=True,
          spat_diag=False, moran=False, white_test=False, vm=False, constant_regi='many',
          cols2regi='all', regime_err_sep=True, cores=False, name_y=None, name_x=None,
          name_regimes=None, name_w=None, name_gwk=None, name_ds=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x, regimes[, w, robust, ...])</code> | Initialize self. |
|--|------------------|

---

## Attributes

---

|                      |
|----------------------|
| <code>mean_y</code>  |
| <code>sig2n</code>   |
| <code>sig2n_k</code> |
| <code>std_y</code>   |
| <code>utu</code>     |
| <code>vm</code>      |

---

### 3.16.2 spreg.ML\_Lag\_Regimes

```
class spreg.ML_Lag_Regimes(y, x, regimes, w=None, constant_regi='many', cols2regi='all',
                           method='full', epsilon=1e-07, regime_lag_sep=False,
                           regime_err_sep=False, cores=False, spat_diag=False, vm=False,
                           name_y=None, name_x=None, name_w=None, name_ds=None,
                           name_regimes=None)
```

ML estimation of the spatial lag model with regimes (note no consistency checks, diagnostics or constants added) [[Ans88](#)].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:
- 'one': a vector of ones is appended to x and held constant across regimes
  - 'many': a vector of ones is appended to x and considered different per regime (default)
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.
- w** [Sparse matrix] Spatial weights sparse matrix
- method** [string] if 'full', brute force calculation (full matrix expressions) if 'ord', Ord eigen-value method if 'LU', LU sparse matrix decomposition

**epsilon** [float] tolerance criterion in minimize\_scalar function and inverse\_product

**regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**spat\_diag** [boolean] if True, include spatial diagnostics (not implemented yet)

**vm** [boolean] if True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

## Examples

Open data baltim.dbf using pysal and create the variables matrices and weights matrix.

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> db = libpysal.io.open(examples.get_path("baltim.dbf"), 'r')
>>> ds_name = "baltim.dbf"
>>> y_name = "PRICE"
>>> y = np.array(db.by_col(y_name)).T
>>> y.shape = (len(y), 1)
>>> x_names = ["NROOM", "AGE", "SQFT"]
>>> x = np.array([db.by_col(var) for var in x_names]).T
>>> ww = ps.open(ps.examples.get_path("baltim_q.gal"))
>>> w = ww.read()
>>> ww.close()
>>> w_name = "baltim_q.gal"
>>> w.transform = 'r'
```

Since in this example we are interested in checking whether the results vary by regimes, we use CITCOU to define whether the location is in the city or outside the city (in the county):

```
>>> regimes = db.by_col("CITCOU")
```

Now we can run the regression with all parameters:

```
>>> mllag = ML_Lag_Regimes(y, x, regimes, w=w, name_y=y_name, name_x=x_names,
→ name_w=w_name, name_ds=ds_name, name_regimes="CITCOU")
>>> np.around(mllag.betas, decimals=4)
array([[ -15.0059],
       [  4.496 ],
       [ -0.0318],
       [  0.35  ],
       [ -4.5404],
       [  3.9219],
       [ -0.1702],
       [  0.8194],
```

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```

[ 0.5385]])
>>> "{0:.6f}".format(mllag.rho)
'0.538503'
>>> "{0:.6f}".format(mllag.mean_y)
'44.307180'
>>> "{0:.6f}".format(mllag.std_y)
'23.606077'
>>> np.around(np.diag(mllag.vml), decimals=4)
array([ 47.42 ,  2.3953,  0.0051,  0.0648,  69.6765,  3.2066,
        0.0116,  0.0486,  0.004 , 390.7274])
>>> np.around(np.diag(mllag.vml), decimals=4)
array([ 47.42 ,  2.3953,  0.0051,  0.0648,  69.6765,  3.2066,
        0.0116,  0.0486,  0.004 ])
>>> "{0:.6f}".format(mllag.sig2)
'200.044334'
>>> "{0:.6f}".format(mllag.logll)
'-864.985056'
>>> "{0:.6f}".format(mllag.aic)
'1747.970112'
>>> "{0:.6f}".format(mllag.schwarz)
'1778.136835'
>>> mllag.title
'MAXIMUM LIKELIHOOD SPATIAL LAG - REGIMES (METHOD = full)'
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] (k+1)x1 array of estimated coefficients (rho first)

**rho** [float] estimate of spatial autoregressive coefficient Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**u** [array] nx1 array of residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant, excluding the rho) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**method** [string] log Jacobian method. if 'full': brute force (full matrix computations) if 'ord', Ord eigenvalue method if 'LU', LU sparse matrix decomposition

**epsilon** [float] tolerance criterion used in minimize\_scalar function and inverse\_product

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (k+1 x k+1), all coefficients



**vm1** [array] Variance covariance matrix ( $k+2 \times k+2$ ), includes sig2 Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**sig2** [float] Sigma squared used in computations Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**logll** [float] maximized log-likelihood (including constant terms) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**aic** [float] Akaike information criterion Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**schwarz** [float] Schwarz criterion Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**predy\_e** [array] predicted values from reduced form

**e\_pred** [array] prediction errors using reduced form predicted values

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form)) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

**title** [string] Name of the regression method used Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**constant\_regi:** ['one', 'many'] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes
- 'many': a vector of ones is appended to x and considered different per regime

**cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.

**regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**regime\_err\_sep: boolean** always set to False - kept for compatibility with other regime models

- kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)
- kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate
- nr** [int] Number of different regimes in the ‘regimes’ list
- multi** [dictionary] Only available when multiple regressions are estimated, i.e. when `regime_err_sep=True` and no variable is fixed across regimes. Contains all attributes of each individual regression

## Methods

### ML\_Lag\_Regimes\_Multi

```
__init__(self, y, x, regimes, w=None, constant_regi='many', cols2regi='all', method='full',
         epsilon=1e-07, regime_lag_sep=False, regime_err_sep=False, cores=False,
         spat_diag=False, vm=False, name_y=None, name_x=None, name_w=None,
         name_ds=None, name_regimes=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

|   |
|---|
| <code>ML_Lag_Regimes_Multi(self, y, x, w_i, w, ...)</code>            |
| <code>__init__(self, y, x, regimes[, w, ...])</code> Initialize self. |

## Attributes

|                      |
|----------------------|
| <code>mean_y</code>  |
| <code>sig2n</code>   |
| <code>sig2n_k</code> |
| <code>std_y</code>   |
| <code>utu</code>     |
| <code>vm</code>      |

### 3.16.3 spreg.ML\_Error\_Regimes

```
class spreg.ML_Error_Regimes(y, x, regimes, w=None, constant_regi='many', cols2regi='all',
                             method='full', epsilon=1e-07, regime_err_sep=False,
                             regime_lag_sep=False, cores=False, spat_diag=False, vm=False,
                             name_y=None, name_x=None, name_w=None, name_ds=None,
                             name_regimes=None)
```

ML estimation of the spatial error model with regimes (note no consistency checks, diagnostics or constants added); Anselin (1988) [[Ans88](#)]

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous)

variable, excluding the constant

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes.
- 'many': a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**w** [Sparse matrix] Spatial weights sparse matrix

**method** [string] if 'full', brute force calculation (full matrix expressions) if 'ord', Ord eigenvalue computation if 'LU', LU sparse matrix decomposition

**epsilon** [float] tolerance criterion in minimize\_scalar function and inverse\_product

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**regime\_lag\_sep: boolean** Always False, kept for consistency in function call, ignored.

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**spat\_diag** [boolean] if True, include spatial diagnostics (not implemented yet)

**vm** [boolean] if True, include variance-covariance matrix in summary results

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

## Examples

Open data baltim.dbf using pysal and create the variables matrices and weights matrix.

```
>>> import numpy as np
>>> import libpysal
>>> db = libpysal.io.open(libpysal.examples.get_path("baltim.dbf"), 'r')
>>> ds_name = "baltim.dbf"
>>> y_name = "PRICE"
>>> y = np.array(db.by_col(y_name)).T
>>> y.shape = (len(y), 1)
>>> x_names = ["NROOM", "AGE", "SQFT"]
>>> x = np.array([db.by_col(var) for var in x_names]).T
>>> ww = ps.open(ps.examples.get_path("baltim_q.gal"))
>>> w = ww.read()
>>> ww.close()
>>> w_name = "baltim_q.gal"
>>> w.transform = 'r'
```

Since in this example we are interested in checking whether the results vary by regimes, we use CITCOU to define whether the location is in the city or outside the city (in the county):

```
>>> regimes = db.by_col("CITCOU")
```

Now we can run the regression with all parameters:

```
>>> mlerr = ML_Error_Regimes(y,x,regimes,w=w,name_y=y_name,name_x=x_names,
    ↪ name_w=w_name,name_ds=ds_name,name_regimes="CITCOU")
>>> np.around(mlerr.betas, decimals=4)
array([[ -2.3949],
       [  4.8738],
       [ -0.0291],
       [  0.3328],
       [ 31.7962],
       [  2.981 ],
       [ -0.2371],
       [  0.8058],
       [  0.6177]])
>>> "{0:.6f}".format(mlerr.lam)
'0.617707'
>>> "{0:.6f}".format(mlerr.mean_y)
'44.307180'
>>> "{0:.6f}".format(mlerr.std_y)
'23.606077'
>>> np.around(mlerr.vml, decimals=4)
array([[ 0.005 , -0.3535],
       [-0.3535, 441.3039]])
>>> np.around(np.diag(mlerr.vml), decimals=4)
array([[ 58.5055,  2.4295,  0.0072,  0.0639,  80.5925,  3.161 ,
         0.012 ,  0.0499,  0.005 ]])
>>> np.around(mlerr.sig2, decimals=4)
array([[ 209.6064]])
>>> "{0:.6f}".format(mlerr.logll)
'-870.333106'
>>> "{0:.6f}".format(mlerr.aic)
'1756.666212'
>>> "{0:.6f}".format(mlerr.schwarz)
'1783.481077'
>>> mlerr.title
'MAXIMUM LIKELIHOOD SPATIAL ERROR - REGIMES (METHOD = full)'
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] (k+1)x1 array of estimated coefficients (lambda last)

**lam** [float] estimate of spatial autoregressive coefficient Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant, excluding the rho) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**method** [string] log Jacobian method. if ‘full’: brute force (full matrix computations) if ‘ord’, Ord eigenvalue computation if ‘LU’, LU sparse matrix decomposition

**epsilon** [float] tolerance criterion used in minimize\_scalar function and inverse\_product

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (k+1 x k+1), all coefficients

**vm1** [array] variance covariance matrix for lambda, sigma (2 x 2) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**sig2** [float] Sigma squared used in computations Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**logll** [float] maximized log-likelihood (including constant terms) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

**title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with ‘x’.

**constant\_regi: string** Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- ‘one’: a vector of ones is appended to x and held constant across regimes.
- ‘many’: a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, ‘all’] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a

list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.

**regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed accross regimes.

**kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the ‘regimes’ list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

## Methods

|                  |  |
|------------------|--|
| <b>get_x_lag</b> |  |
|------------------|--|

```
__init__(self, y, x, regimes, w=None, constant_regi='many', cols2regi='all', method='full',
          epsilon=1e-07, regime_err_sep=False, regime_lag_sep=False, cores=False,
          spat_diag=False, vm=False, name_y=None, name_x=None, name_w=None,
          name_ds=None, name_regimes=None)
```

Initialize self. See help(type(self)) for accurate signature.

## Methods

|   |                  |
|---|------------------|
| <code><b>__init__</b>(self, y, x, regimes[, w, ...])</code> | Initialize self. |
| <code><b>get_x_lag</b>(self, w, regimes_att)</code>         |                  |

## Attributes

|                      |
|----------------------|
| <code>mean_y</code>  |
| <code>sig2n</code>   |
| <code>sig2n_k</code> |
| <code>std_y</code>   |
| <code>utu</code>     |
| <code>vm</code>      |

### 3.16.4 spreg.GM\_Lag\_Regimes

```
class spreg.GM_Lag_Regimes(y, x, regimes, yend=None, q=None, w=None, w_lags=1,
                           lag_q=True, robust=None, gwk=None, sig2n_k=False,
                           spat_diag=False, constant_regi='many', cols2regi='all',
                           regime_lag_sep=False, regime_err_sep=True, cores=False,
                           vm=False, name_y=None, name_x=None, name_yend=None,
                           name_q=None, name_regimes=None, name_w=None,
                           name_gwk=None, name_ds=None)
```

Spatial two stage least squares (S2SLS) with regimes; [Ans88]

#### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x); cannot be used in combination with h

**constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes.
- 'many': a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**w** [pysal W object] Spatial weights object

**w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.

**lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).

**regime\_lag\_sep: boolean** If True (default), the spatial parameter for spatial lag is also computed according to different regimes. If False, the spatial parameter is fixed accross regimes. Option valid only when regime\_err\_sep=True

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**robust** [string] If 'white', then a White consistent estimator of the variance-covariance matrix is given. If 'hac', then a HAC consistent estimator of the variance-covariance matrix is given. If 'ogmm', then Optimal GMM is used to estimate betas and the variance-covariance matrix. Default set to None.

**gwk** [pysal W object] Kernel spatial weights needed for HAC estimation. Note: matrix must have ones along the main diagonal.

**sig2n\_k** [boolean] If True, then use n-k to estimate  $\sigma^2$ . If False, use n.

**spat\_diag** [boolean] If True, then compute Anselin-Kelejian test

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial lag model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.



```
>>> from libpysal import weights
>>> w = weights.Rook.from_shapefile(examples.get_path("NAT.shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

This class runs a lag model, which means that includes the spatial lag of the dependent variable on the right-hand side of the equation. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> model=GM_Lag_Regimes(y, x, regimes, w=w, regime_lag_sep=False, regime_err_
↳sep=False, name_y=y_var, name_x=x_var, name_regimes=r_var, name_ds='NAT', name_
↳w='NAT.shp')
>>> model.betas
array([[ 1.28897623],
       [ 0.79777722],
       [ 0.56366891],
       [ 8.73327838],
       [ 1.30433406],
       [ 0.62418643],
       [-0.39993716]])
```

Once the model is run, we can have a summary of the output by typing: `model.summary` . Alternatively, we can obtain the standard error of the coefficient estimates by calling:

```
>>> model.std_err
array([ 0.44682888,  0.14358192,  0.05655124,  1.06044865,  0.20184548,
        0.06118262,  0.12387232])
```

In the example above, all coefficients but the spatial lag vary according to the regime. It is also possible to have the spatial lag varying according to the regime, which effective will result in an independent spatial lag model estimated for each regime. To run these models, the argument `regime_lag_sep` must be set to `True`:

```
>>> model=GM_Lag_Regimes(y, x, regimes, w=w, regime_lag_sep=True, name_y=y_var,
↳name_x=x_var, name_regimes=r_var, name_ds='NAT', name_w='NAT.shp')
>>> print np.hstack((np.array(model.name_z).reshape(8,1),model.betas,np.
↳sqrt(model.vm.diagonal().reshape(8,1))))
[['0_CONSTANT' '1.36584769' '0.39854720']
 ['0_PS90' '0.80875730' '0.11324884']
 ['0_UE90' '0.56946813' '0.04625087']
 ['0_W_HR90' '-0.4342438' '0.13350159']
 ['1_CONSTANT' '7.90731073' '1.63601874']
 ['1_PS90' '1.27465703' '0.24709870']
 ['1_UE90' '0.60167693' '0.07993322']
 ['1_W_HR90' '-0.2960338' '0.19934459']]
```

Alternatively, we can type: `'model.summary'` to see the organized results output. The class is flexible enough to accommodate a spatial lag model that, besides the spatial lag of the dependent variable, includes other non-spatial endogenous regressors. As an example, we will add the endogenous variable RD90 (resource deprivation) and we decide to instrument for it with FP89 (families below poverty):

```
>>> yd_var = ['RD90']
>>> yd = np.array([db.by_col(name) for name in yd_var]).T
```

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(continued from previous page)

```
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

And we can run the model again:

```
>>> model = GM_Lag_Regimes(y, x, regimes, yend=yd, q=q, w=w, regime_lag_sep=False,
→ regime_err_sep=False, name_y=y_var, name_x=x_var, name_yend=yd_var, name_q=q_
→ var, name_regimes=r_var, name_ds='NAT', name_w='NAT.shp')
>>> model.betas
array([[ 3.42195202],
       [ 1.03311878],
       [ 0.14308741],
       [ 8.99740066],
       [ 1.91877758],
       [-0.32084816],
       [ 2.38918212],
       [ 3.67243761],
       [ 0.06959139]])
```

Once the model is run, we can obtain the standard error of the coefficient estimates. Alternatively, we can have a summary of the output by typing: `model.summary`

```
>>> model.std_err
array([[ 0.49163311,  0.12237382,  0.05633464,  0.72555909,  0.17250521,
         0.06749131,  0.27370369,  0.25106224,  0.05804213]])
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_pred** [array] nx1 array of residuals (using reduced form)

**predy** [array] nx1 array of predicted y values

**predy\_e** [array] nx1 array of predicted y values (using reduced form)

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**kstar** [integer] Number of endogenous variables. Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z** [array] nxk array of variables (combination of x and yend) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**h** [array] nx1 array of instruments (combination of x and q) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**robust** [string] Adjustment for robust standard errors Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form)) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**utu** [float] Sum of squared residuals

**sig2** [float] Sigma squared used in computations Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**ak\_test** [tuple] Anselin-Kelejian test; tuple contains the pair (statistic, p-value) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_gwk** [string] Name of kernel weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

**title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**sig2n** [float] Sigma squared (computed with n in the denominator)

**sig2n\_k** [float] Sigma squared (computed with n-k in the denominator)

**hth** [float]  $H'H$ . Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

- hthi** [float]  $(H'H)^{-1}$ . Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- varb** [array]  $(Z'H(H'H)^{-1}H'Z)^{-1}$ . Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- zthhthi** [array]  $Z'H(H'H)^{-1}$ . Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- pfora1a2** [array]  $n(zthhthi)'varb$  Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- constant\_regi: string** Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:
- 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime.
- cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.
- regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.
- regime\_err\_sep: boolean** If True, a separate regression is run for each regime.
- kr** [int] Number of variables/columns to be "regimized" or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)
- kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate
- nr** [int] Number of different regimes in the 'regimes' list
- multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

## Methods

|                             |  |
|-----------------------------|--|
| <b>GM_Lag_Regimes_Multi</b> |  |
| <b>sp_att_reg</b>           |  |

```
__init__(self, y, x, regimes, yend=None, q=None, w=None, w_lags=1, lag_q=True, robust=None,
          gw_k=None, sig2n_k=False, spat_diag=False, constant_regi='many', cols2regi='all',
          regime_lag_sep=False, regime_err_sep=True, cores=False, vm=False, name_y=None,
          name_x=None, name_yend=None, name_q=None, name_regimes=None, name_w=None,
          name_gwk=None, name_ds=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

|  |                  |
|--|------------------|
| GM_Lag_Regimes_Multi(self, y, x, w_i, w, ...)                  |                  |
| <code>__init__</code> (self, y, x, regimes[, yend, q, w, ...]) | Initialize self. |
| sp_att_reg(self, w_i, regi_ids, wy)                            |                  |

## Attributes

|          |
|----------|
| mean_y   |
| pforala2 |
| sig2n    |
| sig2n_k  |
| std_y    |
| utu      |
| vm       |

### 3.16.5 spreg.GM\_Error\_Regimes

**class** spreg.GM\_Error\_Regimes(y, x, regimes, w, vm=False, name\_y=None, name\_x=None, name\_w=None, constant\_regi='many', cols2regi='all', regime\_err\_sep=False, regime\_lag\_sep=False, cores=False, name\_ds=None, name\_regimes=None)

GMM method for a spatial error model with regimes, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- w** [pysal W object] Spatial weights object
- constant\_regi: string, optional** Switcher controlling the constant term setup. It may take the following values:
  - 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime (default).
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.
- regime\_err\_sep: boolean** If True, a separate regression is run for each regime.
- regime\_lag\_sep: boolean** Always False, kept for consistency, ignored.
- vm** [boolean] If True, include variance-covariance matrix in summary results
- cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output  
**name\_x** [list of strings] Names of independent variables for use in output  
**name\_w** [string] Name of weights matrix for use in output  
**name\_ds** [string] Name of dataset for use in output  
**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely `numpy` to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import libpysal
>>> import numpy as np
```

Open data on NCOVR US County Homicides (3085 areas) using `libpysal.io.open()`. This is the DBF associated with the NAT shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape `(n, 1)` as opposed to the also common shape of `(n, )` that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to `x_var`, such as `x_var = ['Var1', 'Var2', ...]`. Note that PySAL requires this to be an `nxj` numpy array, where `j` is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `NAT.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> model = GM_Error_Regimes(y, x, regimes, w=w, name_y=y_var, name_x=x_var, name_
↳ regimes=r_var, name_ds='NAT.dbf')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. Note that because we are running the classical GMM error model from 1998/99, the spatial parameter is obtained as a point estimate, so although you get a value for it (there are for coefficients under `model.betas`), you cannot perform inference on it (there are only three values in `model.se_betas`). Alternatively, we can have a summary of the output by typing: `model.summary`

```
>>> print model.name_x
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', 'lambda']
>>> np.around(model.betas, decimals=6)
array([[ 0.074807],
       [ 0.786107],
       [ 0.538849],
       [ 5.103756],
       [ 1.196009],
       [ 0.600533],
       [ 0.364103]])
>>> np.around(model.std_err, decimals=6)
array([ 0.379864,  0.152316,  0.051942,  0.471285,  0.19867 ,  0.057252])
>>> np.around(model.z_stat, decimals=6)
array([[ 0.196932,  0.843881],
       [ 5.161042,  0.        ],
       [10.37397 ,  0.        ],
       [10.829455,  0.        ],
       [ 6.02007 ,  0.        ],
       [10.489215,  0.        ]])
>>> np.around(model.sig2, decimals=6)
28.172732
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- predy** [array] nx1 array of predicted y values
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**vm** [array] Variance covariance matrix (kxk)

**sig2** [float] Sigma squared used in computations Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

**title** [string] Name of the regression method used Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**constant\_regi: string** Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes
- 'many': a vector of ones is appended to x and considered different per regime

**cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**kr** [int] Number of variables/columns to be "regimized" or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the 'regimes' list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression



```
__init__(self, y, x, regimes, w, vm=False, name_y=None, name_x=None, name_w=None,
         constant_regi='many', cols2regi='all', regime_err_sep=False, regime_lag_sep=False,
         cores=False, name_ds=None, name_regimes=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, y, x, regimes, w[, vm, ...])</code> | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

### 3.16.6 spreg.GM\_Error\_Het\_Regimes

```
class spreg.GM_Error_Het_Regimes(y, x, regimes, w, max_iter=1, epsilon=1e-05,
                                step1c=False, constant_regi='many', cols2regi='all',
                                regime_err_sep=False, regime_lag_sep=False, cores=False,
                                vm=False, name_y=None, name_x=None, name_w=None,
                                name_ds=None, name_regimes=None)
```

GMM method for a spatial error model with heteroskedasticity and regimes; based on Arraiz et al [ADKP10], following Anselin [Ans11].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- w** [pysal W object] Spatial weights object
- constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:
  - 'one': a vector of ones is appended to x and held constant across regimes
  - 'many': a vector of ones is appended to x and considered different per regime (default)
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.
- regime\_err\_sep: boolean** If True, a separate regression is run for each regime.
- regime\_lag\_sep: boolean** Always False, kept for consistency, ignored.
- max\_iter** [int] Maximum number of iterations of steps 2a and 2b from Arraiz et al. Note: epsilon provides an additional stop condition.
- epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from Arraiz et al. Note: max\_iter provides an additional stop condition.

**step1c** [boolean] If True, then include Step 1c from Arraiz et al.

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Error_Het_Regimes(y, x, regimes, w=w, step1c=True, name_y=y_var,
    ↪name_x=x_var, name_regimes=r_var, name_ds='NAT.dbf')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that explicitly accounts for heteroskedasticity and that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. Alternatively, we can have a summary of the output by typing: `model.summary`

```
>>> print reg.name_x
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', 'lambda']
>>> np.around(reg.betas, decimals=6)
array([[ 0.009121],
       [ 0.812973],
       [ 0.549355],
       [ 5.00279 ],
       [ 1.200929],
       [ 0.614681],
       [ 0.429277]])
>>> np.around(reg.std_err, decimals=6)
array([ 0.355844,  0.221743,  0.059276,  0.686764,  0.35843 ,  0.092788,
        0.02524 ])
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- predy** [array] nx1 array of predicted y values
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [\[ADKP10\]](#).  
Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- iteration** [integer] Number of iterations of steps 2a and 2b from [\[ADKP10\]](#). Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**vm** [array] Variance covariance matrix (kxk)

**sig2** [float] Sigma squared used in computations Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

**title** [string] Name of the regression method used Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**constant\_regi: string** Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes
- 'many': a vector of ones is appended to x and considered different per regime

**cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**kr** [int] Number of variables/columns to be "regimized" or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the 'regimes' list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

---

```
__init__(self, y, x, regimes, w, max_iter=1, epsilon=1e-05, step1c=False, constant_regi='many',
         cols2regi='all', regime_err_sep=False, regime_lag_sep=False, cores=False, vm=False,
         name_y=None, name_x=None, name_w=None, name_ds=None, name_regimes=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

```
__init__(self, y, x, regimes, w[, max_iter, ...])    Initialize self.
```

---

## Attributes

---

```
mean_y
std_y
```

---

### 3.16.7 spreg.GM\_Error\_Hom\_Regimes

```
class spreg.GM_Error_Hom_Regimes(y, x, regimes, w, max_iter=1, epsilon=1e-05, A1='het',
                                cores=False, constant_regi='many', cols2regi='all',
                                regime_err_sep=False, regime_lag_sep=False, vm=False,
                                name_y=None, name_x=None, name_w=None,
                                name_ds=None, name_regimes=None)
```

GMM method for a spatial error model with homoskedasticity, with regimes, results and diagnostics; based on Drukker et al. (2013) [DEP13], following Anselin (2011) [Ans11].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- w** [pysal W object] Spatial weights object
- constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:
  - 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime (default).
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.
- regime\_err\_sep: boolean** If True, a separate regression is run for each regime.
- regime\_lag\_sep: boolean** Always False, kept for consistency, ignored.
- max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.
- epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**A1** [string] If A1='het', then the matrix A1 is defined as in [ADKP10]. If A1='hom', then as in [Ans11]. If A1='hom\_sc', then as in [DEP13] and [DPR13].

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1','Var2',...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial lag model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Error_Hom_Regimes(y, x, regimes, w=w, name_y=y_var, name_x=x_var,
↪name_ds='NAT')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that assumes homoskedasticity but that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. This is why you obtain as many coefficient estimates as standard errors, which you calculate taking the square root of the diagonal of the variance-covariance matrix of the parameters. Alternatively, we can have a summary of the output by typing: `model.summary` `>>> print reg.name_x ['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', 'lambda']`

```
>>> print np.around(reg.betas, 4)
[[ 0.069 ]
 [ 0.7885]
 [ 0.5398]
 [ 5.0948]
 [ 1.1965]
 [ 0.6018]
 [ 0.4104]]
```

```
>>> print np.sqrt(reg.vm.diagonal())
[ 0.39105854  0.15664624  0.05254328  0.48379958  0.20018799  0.05834139
 0.01882401]
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- predy** [array] nx1 array of predicted y values
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10]. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**vm** [array] Variance covariance matrix (kxk)

**sig2** [float] Sigma squared used in computations Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**xtx** [float]  $X'X$ . Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

**title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with ‘x’.

**constant\_regi: string** Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- ‘one’: a vector of ones is appended to x and held constant across regimes.
- ‘many’: a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, ‘all’] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If ‘all’, all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the ‘regimes’ list



**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when `regime_err_sep=True` and no variable is fixed across regimes. Contains all attributes of each individual regression

`__init__`(*self*, *y*, *x*, *regimes*, *w*, *max\_iter=1*, *epsilon=1e-05*, *A1='het'*, *cores=False*, *constant\_regi='many'*, *cols2regi='all'*, *regime\_err\_sep=False*, *regime\_lag\_sep=False*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_w=None*, *name\_ds=None*, *name\_regimes=None*)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__</code> ( <i>self</i> , <i>y</i> , <i>x</i> , <i>regimes</i> , <i>w</i> [, <i>max_iter</i> , ...]) | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

### 3.16.8 spreg.GM\_Combo\_Regimes

**class** `spreg.GM_Combo_Regimes`(*y*, *x*, *regimes*, *yend=None*, *q=None*, *w=None*, *w\_lags=1*, *lag\_q=True*, *cores=False*, *constant\_regi='many'*, *cols2regi='all'*, *regime\_err\_sep=False*, *regime\_lag\_sep=False*, *vm=False*, *name\_y=None*, *name\_x=None*, *name\_yend=None*, *name\_q=None*, *name\_w=None*, *name\_ds=None*, *name\_regimes=None*)

GMM method for a spatial lag and error model with regimes and endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)
- w** [pysal W object] Spatial weights object (always needed)
- constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:
  - 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime (default).
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.

**lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial lag model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

The Combo class runs an SARAR model, that is a spatial lag+error model. In this case we will run a simple version of that, where we have the spatial effects as well as exogenous variables. Since it is a spatial model, we have to pass in the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> model = GM_Combo_Regimes(y, x, regimes, w=w, name_y=y_var, name_x=x_var, name_
↪regimes=r_var, name_ds='NAT')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. Note that because we are running the classical GMM error model from 1998/99, the spatial parameter is obtained as a point estimate, so although you get a value for it (there are for coefficients under model.betas), you cannot perform inference on it (there are only three values in model.se\_betas). Also, this regression uses a two stage least squares estimation method that accounts for the endogeneity created by the spatial lag of the dependent variable. We can have a summary of the output by typing: model.summary Alternatively, we can check the betas:

```
>>> print model.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '_Global_W_
↪HR90', 'lambda']
>>> print np.around(model.betas,4)
[[ 1.4607]
 [ 0.958 ]
 [ 0.5658]
 [ 9.113 ]
 [ 1.1338]
 [ 0.6517]
 [-0.4583]
 [ 0.6136]]
```

And lambda:

```
>>> print 'lambda: ', np.around(model.betas[-1], 4)
lambda: [ 0.6136]
```

This class also allows the user to run a spatial lag+error model with the extra feature of including non-spatial endogenous regressors. This means that, in addition to the spatial lag and error, we consider some of the variables on the right-hand side of the equation as endogenous and we instrument for this. In this case we consider RD90 (resource deprivation) as an endogenous regressor. We use FP89 (families below poverty) for this and hence put it in the instruments parameter, 'q'.

```
>>> yd_var = ['RD90']
>>> yd = np.array([db.by_col(name) for name in yd_var]).T
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

And then we can run and explore the model analogously to the previous combo:

```
>>> model = GM_Combo_Regimes(y, x, regimes, yd, q, w=w, name_y=y_var, name_x=x_
↳var, name_yend=yd_var, name_q=q_var, name_regimes=r_var, name_ds='NAT')
>>> print model.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '0_RD90', '1_
↳RD90', '_Global_W_HR90', 'lambda']
>>> print model.betas
[[ 3.41963782]
 [ 1.04065841]
 [ 0.16634393]
 [ 8.86544628]
 [ 1.85120528]
 [-0.24908469]
 [ 2.43014046]
 [ 3.61645481]
 [ 0.03308671]
 [ 0.18684992]]
>>> print np.sqrt(model.vcov.diagonal())
[ 0.53067577  0.13271426  0.06058025  0.76406411  0.17969783  0.07167421
  0.28943121  0.25308326  0.06126529]
>>> print 'lambda: ', np.around(model.betas[-1], 4)
lambda: [ 0.1868]
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- e\_pred** [array] nx1 array of residuals (using reduced form)
- predy** [array] nx1 array of predicted y values
- predy\_e** [array] nx1 array of predicted y values (using reduced form)
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable  
Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- z** [array] nxk array of variables (combination of x and yend) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form)) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**sig2** [float] Sigma squared used in computations (based on filtered residuals) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

**title** [string] Name of the regression method used Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**constant\_regi** [string] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes.
- 'many': a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the ‘regimes’ list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when `regime_err_sep=True` and no variable is fixed across regimes. Contains all attributes of each individual regression

```
__init__(self, y, x, regimes, yend=None, q=None, w=None, w_lags=1, lag_q=True, cores=False,
         constant_regi='many', cols2regi='all', regime_err_sep=False, regime_lag_sep=False,
         vm=False, name_y=None, name_x=None, name_yend=None, name_q=None,
         name_w=None, name_ds=None, name_regimes=None)
```

Initialize self. See `help(type(self))` for accurate signature.

## Methods

---

|   |                  |
|---|------------------|
| <code>__init__(self, y, x, regimes[, yend, q, w, ...])</code> | Initialize self. |
|---|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

### 3.16.9 spreg.GM\_Combo\_Hom\_Regimes

```
class spreg.GM_Combo_Hom_Regimes(y, x, regimes, yend=None, q=None, w=None, w_lags=1,
                                  lag_q=True, cores=False, max_iter=1, epsilon=1e-05,
                                  A1='het', constant_regi='many', cols2regi='all',
                                  regime_err_sep=False, regime_lag_sep=False, vm=False,
                                  name_y=None, name_x=None, name_yend=None,
                                  name_q=None, name_w=None, name_ds=None,
                                  name_regimes=None)
```

GMM method for a spatial lag and error model with homoskedasticity, regimes and endogenous variables, with results and diagnostics; based on Drukker et al. (2013) [DEP13], following Anselin (2011) [Ans11].

#### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with ‘x’.

**w** [pysal W object] Spatial weights object (always needed)

**constant\_regi:** **string** Switcher controlling the constant term setup. It may take the following values:

- ‘one’: a vector of ones is appended to x and held constant across regimes.
- ‘many’: a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, ‘all’] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If ‘all’ (default), all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**regime\_lag\_sep** [boolean] If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.

**lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**A1** [string] If A1=‘het’, then the matrix A1 is defined as in [ADKP10]. If A1=‘hom’, then as in [Ans11]. If A1=‘hom\_sc’, then as in [DEP13] and [DPR13].

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial combo model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

Example only with spatial lag

The Combo class runs an SARAR model, that is a spatial lag+error model. In this case we will run a simple version of that, where we have the spatial effects as well as exogenous variables. Since it is a spatial model, we have to pass in the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional. We can have a summary of the output by typing: model.summary Alternatively, we can check the betas:

```
>>> reg = GM_Combo_Hom_Regimes(y, x, regimes, w=w, A1='hom_sc', name_y=y_var,
↪name_x=x_var, name_regimes=r_var, name_ds='NAT')
>>> print reg.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '_Global_W_
↪HR90', 'lambda']
>>> print np.around(reg.betas,4)
[[ 1.4607]
 [ 0.9579]]
```

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```
[ 0.5658]
[ 9.1129]
[ 1.1339]
[ 0.6517]
[-0.4583]
[ 0.6634]]
```

This class also allows the user to run a spatial lag+error model with the extra feature of including non-spatial endogenous regressors. This means that, in addition to the spatial lag and error, we consider some of the variables on the right-hand side of the equation as endogenous and we instrument for this. In this case we consider RD90 (resource deprivation) as an endogenous regressor. We use FP89 (families below poverty) for this and hence put it in the instruments parameter, 'q'.

```
>>> yd_var = ['RD90']
>>> yd = np.array([db.by_col(name) for name in yd_var]).T
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

And then we can run and explore the model analogously to the previous combo:

```
>>> reg = GM_Combo_Hom_Regimes(y, x, regimes, yd, q, w=w, A1='hom_sc', name_y=y_
↳var, name_x=x_var, name_yend=yd_var, name_q=q_var, name_regimes=r_var, name_ds=
↳'NAT')
>>> print reg.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '0_RD90', '1_
↳RD90', '_Global_W_HR90', 'lambda']
>>> print reg.betas
[[ 3.4196478 ]
[ 1.04065595]
[ 0.16630304]
[ 8.86570777]
[ 1.85134286]
[-0.24921597]
[ 2.43007651]
[ 3.61656899]
[ 0.03315061]
[ 0.22636055]]
>>> print np.sqrt(reg.vm.diagonal())
[ 0.53989913  0.13506086  0.06143434  0.77049956  0.18089997  0.07246848
 0.29218837  0.25378655  0.06184801  0.06323236]
>>> print 'lambda: ', np.around(reg.betas[-1], 4)
lambda: [ 0.2264]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**e\_pred** [array] nx1 array of residuals (using reduced form)

**predy** [array] nx1 array of predicted y values

**predy\_e** [array] nx1 array of predicted y values (using reduced form)

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z** [array] nxk array of variables (combination of x and yend) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**h** [array] nx1 array of instruments (combination of x and q) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10]. Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form)) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**sig2** [float] Sigma squared used in computations (based on filtered residuals) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

**title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with ‘x’.

**constant\_regi** [string] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- ‘one’: a vector of ones is appended to x and held constant across regimes.
- ‘many’: a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, ‘all’] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If ‘all’, all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**regime\_lag\_sep** [boolean] If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the ‘regimes’ list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

**\_\_init\_\_** (*self, y, x, regimes, yend=None, q=None, w=None, w\_lags=1, lag\_q=True, cores=False, max\_iter=1, epsilon=1e-05, A1='het', constant\_regi='many', cols2regi='all', regime\_err\_sep=False, regime\_lag\_sep=False, vm=False, name\_y=None, name\_x=None, name\_yend=None, name\_q=None, name\_w=None, name\_ds=None, name\_regimes=None*)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|   |                  |
|---|------------------|
| <code>__init__(self, y, x, regimes[, yend, q, w, ...])</code> | Initialize self. |
|---|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

### 3.16.10 spreg.GM\_Combo\_Het\_Regimes

```
class spreg.GM_Combo_Het_Regimes(y, x, regimes, yend=None, q=None, w=None,
                                w_lags=1, lag_q=True, max_iter=1, epsilon=1e-05,
                                step1c=False, cores=False, inv_method='power_exp', con-
                                stant_regi='many', cols2regi='all', regime_err_sep=False,
                                regime_lag_sep=False, vm=False, name_y=None,
                                name_x=None, name_yend=None, name_q=None,
                                name_w=None, name_ds=None, name_regimes=None)
```

GMM method for a spatial lag and error model with heteroskedasticity, regimes and endogenous variables, with results and diagnostics; based on Arraiz et al [ADKP10], following Anselin [Ans11].

#### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**w** [pysal W object] Spatial weights object (always needed)

**constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes.
- 'many': a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**regime\_lag\_sep** [boolean] If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.

**w\_lags** [integer] Orders of W to include as instruments for the spatially lagged dependent variable. For example, w\_lags=1, then instruments are WX; if w\_lags=2, then WX, WWX; and so on.

**lag\_q** [boolean] If True, then include spatial lags of the additional instruments (q).

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**step1c** [boolean] If True, then include Step 1c from [ADKP10].

**inv\_method** [string] If "power\_exp", then compute inverse using the power expansion. If "true\_inv", then compute the true inverse. Note that true\_inv will fail for large n.

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial combo model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

Example only with spatial lag

The Combo class runs an SARAR model, that is a spatial lag+error model. In this case we will run a simple version of that, where we have the spatial effects as well as exogenous variables. Since it is a spatial model, we have to pass in the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional. We can have a summary of the output by typing: `model.summary` Alternatively, we can check the betas:

```
>>> reg = GM_Combo_Het_Regimes(y, x, regimes, w=w, step1c=True, name_y=y_var,
↳ name_x=x_var, name_regimes=r_var, name_ds='NAT')
>>> print reg.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '_Global_W_
↳ HR90', 'lambda']
>>> print np.around(reg.betas,4)
[[ 1.4613]
 [ 0.9587]
 [ 0.5658]
 [ 9.1157]
 [ 1.1324]
 [ 0.6518]
 [-0.4587]
 [ 0.7174]]
```

This class also allows the user to run a spatial lag+error model with the extra feature of including non-spatial endogenous regressors. This means that, in addition to the spatial lag and error, we consider some of the variables on the right-hand side of the equation as endogenous and we instrument for this. In this case we consider RD90 (resource deprivation) as an endogenous regressor. We use FP89 (families below poverty) for this and hence put it in the instruments parameter, 'q'.

```
>>> yd_var = ['RD90']
>>> yd = np.array([db.by_col(name) for name in yd_var]).T
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

And then we can run and explore the model analogously to the previous combo:

```
>>> reg = GM_Combo_Het_Regimes(y, x, regimes, yd, q, w=w, step1c=True, name_y=y_
↳ var, name_x=x_var, name_yend=yd_var, name_q=q_var, name_regimes=r_var, name_ds=
↳ 'NAT')
>>> print reg.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '0_RD90', '1_
↳ RD90', '_Global_W_HR90', 'lambda']
>>> print reg.betas
[[ 3.41936197]
 [ 1.04071048]
 [ 0.16747219]
 [ 8.85820215]
 [ 1.847382 ]
```

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```

[-0.24545394]
[ 2.43189808]
[ 3.61328423]
[ 0.03132164]
[ 0.29544224]]
>>> print np.sqrt(reg.vm.diagonal())
[ 0.53103804  0.20835827  0.05755679  1.00496234  0.34332131  0.10259525
 0.3454436   0.37932794  0.07611667  0.07067059]
>>> print 'lambda: ', np.around(reg.betas[-1], 4)
lambda: [ 0.2954]

```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**e\_pred** [array] nx1 array of residuals (using reduced form)

**predy** [array] nx1 array of predicted y values

**predy\_e** [array] nx1 array of predicted y values (using reduced form)

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z** [array] nxk array of variables (combination of x and yend) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**h** [array] nx1 array of instruments (combination of x and q) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10].  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10]. Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix (kxk)

- pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- pr2\_e** [float] Pseudo R squared (squared correlation between y and ypred\_e (using reduced form)) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_yend** [list of strings] Names of endogenous variables for use in output
- name\_z** [list of strings] Names of exogenous and endogenous variables for use in output
- name\_q** [list of strings] Names of external instruments
- name\_h** [list of strings] Names of all instruments used in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output
- name\_regimes** [string] Name of regimes variable for use in output
- title** [string] Name of the regression method used Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- constant\_regi** [string] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:
- 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime (default).
- cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.
- regime\_err\_sep: boolean** If True, a separate regression is run for each regime.
- regime\_lag\_sep: boolean** If True, the spatial parameter for spatial lag is also computed according to different regimes. If False (default), the spatial parameter is fixed across regimes.
- kr** [int] Number of variables/columns to be "regimized" or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)
- kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate
- nr** [int] Number of different regimes in the 'regimes' list



**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when `regime_err_sep=True` and no variable is fixed across regimes. Contains all attributes of each individual regression

```
__init__(self, y, x, regimes, yend=None, q=None, w=None, w_lags=1, lag_q=True,
          max_iter=1, epsilon=1e-05, step1c=False, cores=False, inv_method='power_exp',
          constant_regi='many', cols2regi='all', regime_err_sep=False, regime_lag_sep=False,
          vm=False, name_y=None, name_x=None, name_yend=None, name_q=None,
          name_w=None, name_ds=None, name_regimes=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|   |                  |
|---|------------------|
| <code>__init__(self, y, x, regimes[, yend, q, w, ...])</code> | Initialize self. |
|---|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

### 3.16.11 spreg.GM\_Endog\_Error\_Regimes

```
class spreg.GM_Endog_Error_Regimes(y, x, yend, q, regimes, w, cores=False,
                                   vm=False, constant_regi='many', cols2regi='all',
                                   regime_err_sep=False, regime_lag_sep=False,
                                   name_y=None, name_x=None, name_yend=None,
                                   name_q=None, name_w=None, name_ds=None,
                                   name_regimes=None, summ=True, add_lag=False)
```

GMM method for a spatial error model with regimes and endogenous variables, with results and diagnostics; based on Kelejian and Prucha (1998, 1999) [KP98] [KP99].

#### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- w** [pysal W object] Spatial weights object
- constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:
  - 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime (default).
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating

for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**regime\_lag\_sep: boolean** Always False, kept for consistency, ignored.

**vm** [boolean] If True, include variance-covariance matrix in summary results

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import libpysal
>>> import numpy as np
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...] Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

For the endogenous models, we add the endogenous variable RD90 (resource deprivation) and we decide to instrument for it with FP89 (families below poverty):

```
>>> yd_var = ['RD90']
>>> yend = np.array([db.by_col(name) for name in yd_var]).T
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↳"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous), the instruments and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> model = GM_Endog_Error_Regimes(y, x, yend, q, regimes, w=w, name_y=y_var,
↳name_x=x_var, name_yend=yd_var, name_q=q_var, name_regimes=r_var, name_ds='NAT.
↳dbf')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. Note that because we are running the classical GMM error model from 1998/99, the spatial parameter is obtained as a point estimate, so although you get a value for it (there are for coefficients under model.betas), you cannot perform inference on it (there are only three values in model.se\_betas). Also, this regression uses a two stage least squares estimation method that accounts for the endogeneity created by the endogenous variables included. Alternatively, we can have a summary of the output by typing: model.summary

```
>>> print model.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '0_RD90', '1_
↳RD90', 'lambda']
>>> np.around(model.betas, decimals=5)
array([[ 3.59718],
       [ 1.0652 ],
       [ 0.15822],
       [ 9.19754],
       [ 1.88082],
       [-0.24878],
       [ 2.46161],
       [ 3.57943],
       [ 0.25564]])
>>> np.around(model.std_err, decimals=6)
array([ 0.522633,  0.137555,  0.063054,  0.473654,  0.18335 ,  0.072786,
        0.300711,  0.240413])
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array]  $k \times 1$  array of estimated coefficients

**u** [array]  $n \times 1$  array of residuals

**e\_filtered** [array]  $n \times 1$  array of spatially filtered residuals

**predy** [array]  $n \times 1$  array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**y** [array]  $n \times 1$  array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable  
Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z** [array]  $n \times k$  array of variables (combination of x and yend) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**mean\_y** [float] Mean of dependent variable

**std\_y** [float] Standard deviation of dependent variable

**vm** [array] Variance covariance matrix ( $k \times k$ )

**pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**sig2** [float] Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details) Sigma squared used in computations

**std\_err** [array]  $1 \times k$  array of standard errors of the betas Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary 'multi' when multiple regressions (see 'multi' below for details)

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_z** [list of strings] Names of exogenous and endogenous variables for use in output

**name\_q** [list of strings] Names of external instruments

**name\_h** [list of strings] Names of all instruments used in output

**name\_w** [string] Name of weights matrix for use in output

**name\_ds** [string] Name of dataset for use in output

**name\_regimes** [string] Name of regimes variable for use in output

**title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with ‘x’.

**constant\_regi** [['one', 'many']] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- ‘one’: a vector of ones is appended to x and held constant across regimes.
- ‘many’: a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, ‘all’] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If ‘all’, all the variables vary by regime.

**regime\_err\_sep: boolean** If True, a separate regression is run for each regime.

**kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the ‘regimes’ list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

```
__init__(self, y, x, yend, q, regimes, w, cores=False, vm=False, constant_regi='many',
         cols2regi='all', regime_err_sep=False, regime_lag_sep=False, name_y=None,
         name_x=None, name_yend=None, name_q=None, name_w=None, name_ds=None,
         name_regimes=None, summ=True, add_lag=False)
```

Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|   |                  |
|---|------------------|
| <code>__init__(self, y, x, yend, q, regimes, w[, ...])</code> | Initialize self. |
|---|------------------|

---

## Attributes

---

|                     |  |
|---------------------|--|
| <code>mean_y</code> |  |
| <code>std_y</code>  |  |

---

### 3.16.12 spreg.GM\_Endog\_Error\_Hom\_Regimes

```
class spreg.GM_Endog_Error_Hom_Regimes(y, x, yend, q, regimes, w, constant_regi='many',
                                         cols2regi='all', regime_err_sep=False,
                                         regime_lag_sep=False, max_iter=1, epsilon=1e-
05, A1='het', cores=False, vm=False,
                                         name_y=None, name_x=None, name_yend=None,
                                         name_q=None, name_w=None, name_ds=None,
                                         name_regimes=None, summ=True, add_lag=False)
```

GMM method for a spatial error model with homoskedasticity, regimes and endogenous variables. Based on Drukker et al. (2013) [DEP13], following Anselin (2011) [Ans11].

#### Parameters

**y** [array] nx1 array for dependent variable

**x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant

**yend** [array] Two dimensional array with n rows and one column for each endogenous variable

**q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**w** [pysal W object] Spatial weights object

**constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes.
- 'many': a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**regime\_lag\_sep** [boolean] Always False, kept for consistency, ignored.

**max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.

**epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.

**A1** [string] If A1='het', then the matrix A1 is defined as in [ADKP10]. If A1='hom', then as in [Ans11]. If A1='hom\_sc', then as in [DEP13] and [DPR13].

**cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.

**name\_y** [string] Name of dependent variable for use in output

**name\_x** [list of strings] Names of independent variables for use in output

**name\_yend** [list of strings] Names of endogenous variables for use in output

**name\_q** [list of strings] Names of instruments for use in output

**name\_w** [string] Name of weights matrix for use in output  
**name\_ds** [string] Name of dataset for use in output  
**name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely numpy to convert the data we read into arrays that spreg understands and pysal to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to x\_var, such as x\_var = ['Var1', 'Var2', ...]. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

For the endogenous models, we add the endogenous variable RD90 (resource deprivation) and we decide to instrument for it with FP89 (families below poverty):

```
>>> yd_var = ['RD90']
>>> yend = np.array([db.by_col(name) for name in yd_var]).T
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from NAT.shp.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous), the instruments and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Endog_Error_Hom_Regimes(y, x, yend, q, regimes, w=w, A1='hom_sc',  
↳ name_y=y_var, name_x=x_var, name_yend=yd_var, name_q=q_var, name_regimes=r_var,  
↳ name_ds='NAT.dbf')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that assumes homoskedasticity but that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. Hence, we find the same number of betas as of standard errors, which we calculate taking the square root of the diagonal of the variance-covariance matrix. Alternatively, we can have a summary of the output by typing: `model.summary`

```
>>> print reg.name_z  
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '0_RD90', '1_  
↳ RD90', 'lambda']
```

```
>>> print np.around(reg.betas,4)  
[[ 3.5973]  
 [ 1.0652]  
 [ 0.1582]  
 [ 9.198 ]  
 [ 1.8809]  
 [-0.2489]  
 [ 2.4616]  
 [ 3.5796]  
 [ 0.2541]]
```

```
>>> print np.around(np.sqrt(reg.vm.diagonal()),4)  
[ 0.5204  0.1371  0.0629  0.4721  0.1824  0.0725  0.2992  0.2395  0.024 ]
```

### Attributes

**summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)

**betas** [array] kx1 array of estimated coefficients

**u** [array] nx1 array of residuals

**e\_filtered** [array] nx1 array of spatially filtered residuals

**predy** [array] nx1 array of predicted y values

**n** [integer] Number of observations

**k** [integer] Number of variables for which coefficients are estimated (including the constant)  
Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**y** [array] nx1 array for dependent variable



- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable used as instruments Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- z** [array] nxk array of variables (combination of x and yend) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- h** [array] nxl array of instruments (combination of x and q) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [\[ADKP10\]](#). Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- iteration** [integer] Number of iterations of steps 2a and 2b from [\[ADKP10\]](#). Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- mean\_y** [float] Mean of dependent variable
- std\_y** [float] Standard deviation of dependent variable
- vm** [array] Variance covariance matrix (kxk)
- pr2** [float] Pseudo R squared (squared correlation between y and ypred) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- sig2** [float] Sigma squared used in computations Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- std\_err** [array] 1xk array of standard errors of the betas Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- z\_stat** [list of tuples] z statistic; each tuple contains the pair (statistic, p-value), where each is a float Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- hth** [float]  $H'H$ . Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_yend** [list of strings] Names of endogenous variables for use in output
- name\_z** [list of strings] Names of exogenous and endogenous variables for use in output
- name\_q** [list of strings] Names of external instruments
- name\_h** [list of strings] Names of all instruments used in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output
- name\_regimes** [string] Name of regimes variable for use in output
- title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**constant\_regi** [['one', 'many']] Ignored if regimes=False. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:

- 'one': a vector of ones is appended to x and held constant across regimes.
- 'many': a vector of ones is appended to x and considered different per regime (default).

**cols2regi** [list, 'all'] Ignored if regimes=False. Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all', all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**kr** [int] Number of variables/columns to be "regimized" or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the 'regimes' list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

**\_\_init\_\_** (self, y, x, yend, q, regimes, w, constant\_regi='many', cols2regi='all', regime\_err\_sep=False, regime\_lag\_sep=False, max\_iter=1, epsilon=1e-05, A1='het', cores=False, vm=False, name\_y=None, name\_x=None, name\_yend=None, name\_q=None, name\_w=None, name\_ds=None, name\_regimes=None, summ=True, add\_lag=False)  
Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__</code> (self, y, x, yend, q, regimes, w[, ...]) | Initialize self. |
|--|------------------|

---

## Attributes

---

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

---

### 3.16.13 spreg.GM\_Endog\_Error\_Het\_Regimes

**class** spreg.GM\_Endog\_Error\_Het\_Regimes (y, x, yend, q, regimes, w, max\_iter=1, epsilon=1e-05, step1c=False, constant\_regi='many', cols2regi='all', regime\_err\_sep=False, regime\_lag\_sep=False, inv\_method='power\_exp', cores=False, vm=False, name\_y=None, name\_x=None, name\_yend=None, name\_q=None, name\_w=None, name\_ds=None, name\_regimes=None, summ=True, add\_lag=False)

GMM method for a spatial error model with heteroskedasticity, regimes and endogenous variables, with results

and diagnostics; based on Arraiz et al [ADKP10], following Anselin [Ans11].

### Parameters

- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, excluding the constant
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable
- q** [array] Two dimensional array with n rows and one column for each external exogenous variable to use as instruments (note: this should not contain any variables from x)
- regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.
- w** [pysal W object] Spatial weights object
- constant\_regi: string** Switcher controlling the constant term setup. It may take the following values:
  - 'one': a vector of ones is appended to x and held constant across regimes.
  - 'many': a vector of ones is appended to x and considered different per regime (default).
- cols2regi** [list, 'all'] Argument indicating whether each column of x should be considered as different per regime or held constant across regimes (False). If a list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If 'all' (default), all the variables vary by regime.
- regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.
- regime\_lag\_sep** [boolean] Always False, kept for consistency, ignored.
- max\_iter** [int] Maximum number of iterations of steps 2a and 2b from [ADKP10]. Note: epsilon provides an additional stop condition.
- epsilon** [float] Minimum change in lambda required to stop iterations of steps 2a and 2b from [ADKP10]. Note: max\_iter provides an additional stop condition.
- step1c** [boolean] If True, then include Step 1c from [ADKP10].
- inv\_method** [string] If "power\_exp", then compute inverse using the power expansion. If "true\_inv", then compute the true inverse. Note that true\_inv will fail for large n.
- vm** [boolean] If True, include variance-covariance matrix in summary results
- cores** [boolean] Specifies if multiprocessing is to be used Default: no multiprocessing, cores = False Note: Multiprocessing may not work on all platforms.
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_yend** [list of strings] Names of endogenous variables for use in output
- name\_q** [list of strings] Names of instruments for use in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output
- name\_regimes** [string] Name of regime variable for use in the output

## Examples

We first need to import the needed modules, namely `numpy` to convert the data we read into arrays that `spreg` understands and `pysal` to perform all the analysis.

```
>>> import numpy as np
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using `libpysal.io.open()`. This is the DBF associated with the NAT shapefile. Note that `libpysal.io.open()` also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

Extract the HR90 column (homicide rates in 1990) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape  $(n, 1)$  as opposed to the also common shape of  $(n, )$  that other packages accept.

```
>>> y_var = 'HR90'
>>> y = np.array([db.by_col(y_var)]).reshape(3085,1)
```

Extract UE90 (unemployment rate) and PS90 (population structure) vectors from the DBF to be used as independent variables in the regression. Other variables can be inserted by adding their names to `x_var`, such as `x_var = ['Var1', 'Var2', ...]`. Note that PySAL requires this to be an  $n \times j$  numpy array, where  $j$  is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in.

```
>>> x_var = ['PS90', 'UE90']
>>> x = np.array([db.by_col(name) for name in x_var]).T
```

For the endogenous models, we add the endogenous variable RD90 (resource deprivation) and we decide to instrument for it with FP89 (families below poverty):

```
>>> yd_var = ['RD90']
>>> yend = np.array([db.by_col(name) for name in yd_var]).T
>>> q_var = ['FP89']
>>> q = np.array([db.by_col(name) for name in q_var]).T
```

The different regimes in this data are given according to the North and South dummy (SOUTH).

```
>>> r_var = 'SOUTH'
>>> regimes = db.by_col(r_var)
```

Since we want to run a spatial error model, we need to specify the spatial weights matrix that includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing `gal` file or create a new one. In this case, we will create one from `NAT.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are all set with the preliminaries, we are good to run the model. In this case, we will need the variables (exogenous and endogenous), the instruments and the weights matrix. If we want to have the names of the variables printed in the output summary, we will have to pass them in as well, although this is optional.

```
>>> reg = GM_Endog_Error_Het_Regimes(y, x, yend, q, regimes, w=w, step1c=True,
↳name_y=y_var, name_x=x_var, name_yend=yd_var, name_q=q_var, name_regimes=r_var,
↳name_ds='NAT.dbf')
```

Once we have run the model, we can explore a little bit the output. The regression object we have created has many attributes so take your time to discover them. This class offers an error model that explicitly accounts for heteroskedasticity and that unlike the models from `spreg.error_sp`, it allows for inference on the spatial parameter. Hence, we find the same number of betas as of standard errors, which we calculate taking the square root of the diagonal of the variance-covariance matrix. Alternatively, we can have a summary of the output by typing: `model.summary`

```
>>> print reg.name_z
['0_CONSTANT', '0_PS90', '0_UE90', '1_CONSTANT', '1_PS90', '1_UE90', '0_RD90', '1_
↳RD90', 'lambda']
```

```
>>> print np.around(reg.betas,4)
[[ 3.5944]
 [ 1.065 ]
 [ 0.1587]
 [ 9.184 ]
 [ 1.8784]
 [-0.2466]
 [ 2.4617]
 [ 3.5756]
 [ 0.2908]]
```

```
>>> print np.around(np.sqrt(reg.vm.diagonal()),4)
[ 0.5043  0.2132  0.0581  0.6681  0.3504  0.0999  0.3686  0.3402  0.028 ]
```

### Attributes

- summary** [string] Summary of regression results and diagnostics (note: use in conjunction with the print command)
- betas** [array] kx1 array of estimated coefficients
- u** [array] nx1 array of residuals
- e\_filtered** [array] nx1 array of spatially filtered residuals
- predy** [array] nx1 array of predicted y values
- n** [integer] Number of observations
- k** [integer] Number of variables for which coefficients are estimated (including the constant). Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- y** [array] nx1 array for dependent variable
- x** [array] Two dimensional array with n rows and one column for each independent (exogenous) variable, including the constant. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- yend** [array] Two dimensional array with n rows and one column for each endogenous variable. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)

- q** [array] Two dimensional array with  $n$  rows and one column for each external exogenous variable used as instruments Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- z** [array]  $n \times k$  array of variables (combination of  $x$  and  $yend$ ) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- h** [array]  $n \times l$  array of instruments (combination of  $x$  and  $q$ ) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- iter\_stop** [string] Stop criterion reached during iteration of steps 2a and 2b from [ADKP10]. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- iteration** [integer] Number of iterations of steps 2a and 2b from [ADKP10]. Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- mean\_y** [float] Mean of dependent variable
- std\_y** [float] Standard deviation of dependent variable
- vm** [array] Variance covariance matrix ( $k \times k$ )
- pr2** [float] Pseudo R squared (squared correlation between  $y$  and  $ypred$ ) Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- std\_err** [array]  $1 \times k$  array of standard errors of the betas Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- z\_stat** [list of tuples]  $z$  statistic; each tuple contains the pair (statistic,  $p$ -value), where each is a float Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- name\_y** [string] Name of dependent variable for use in output
- name\_x** [list of strings] Names of independent variables for use in output
- name\_yend** [list of strings] Names of endogenous variables for use in output
- name\_z** [list of strings] Names of exogenous and endogenous variables for use in output
- name\_q** [list of strings] Names of external instruments
- name\_h** [list of strings] Names of all instruments used in output
- name\_w** [string] Name of weights matrix for use in output
- name\_ds** [string] Name of dataset for use in output
- name\_regimes** [string] Name of regimes variable for use in output
- title** [string] Name of the regression method used Only available in dictionary ‘multi’ when multiple regressions (see ‘multi’ below for details)
- regimes** [list] List of  $n$  values with the mapping of each observation to a regime. Assumed to be aligned with ‘ $x$ ’.
- constant\_regi** [string] Ignored if `regimes=False`. Constant option for regimes. Switcher controlling the constant term setup. It may take the following values:
  - ‘one’: a vector of ones is appended to  $x$  and held constant across regimes.
  - ‘many’: a vector of ones is appended to  $x$  and considered different per regime (default).
- cols2regi** [list, ‘all’] Ignored if `regimes=False`. Argument indicating whether each column of  $x$  should be considered as different per regime or held constant across regimes (False). If a

list, k booleans indicating for each variable the option (True if one per regime, False to be held constant). If ‘all’, all the variables vary by regime.

**regime\_err\_sep** [boolean] If True, a separate regression is run for each regime.

**kr** [int] Number of variables/columns to be “regimized” or subject to change by regime. These will result in one parameter estimate by regime for each variable (i.e. nr parameters per variable)

**kf** [int] Number of variables/columns to be considered fixed or global across regimes and hence only obtain one parameter estimate

**nr** [int] Number of different regimes in the ‘regimes’ list

**multi** [dictionary] Only available when multiple regressions are estimated, i.e. when regime\_err\_sep=True and no variable is fixed across regimes. Contains all attributes of each individual regression

```
__init__(self, y, x, yend, q, regimes, w, max_iter=1, epsilon=1e-05, step1c=False, constant_regi='many', cols2regi='all', regime_err_sep=False, regime_lag_sep=False, inv_method='power_exp', cores=False, vm=False, name_y=None, name_x=None, name_yend=None, name_q=None, name_w=None, name_ds=None, name_regimes=None, summ=True, add_lag=False)
```

Initialize self. See help(type(self)) for accurate signature.

## Methods

|   |                  |
|---|------------------|
| <code>__init__(self, y, x, yend, q, regimes, w[, ...])</code> | Initialize self. |
|---|------------------|

## Attributes

|                     |
|---------------------|
| <code>mean_y</code> |
| <code>std_y</code>  |

## 3.17 Seemingly-Unrelated Regressions

Seemingly-unrelated regression models are a generalization of linear regression. These models (and their spatial generalizations) allow for correlation in the residual terms between groups that use the same model. In spatial Seemingly-Unrelated Regressions, the error terms across groups are allowed to exhibit a structured type of correlation: spatial correlation.

|   |  |
|---|--|
| <code>spreg.SUR(bigy, bigX[, w, regimes, ...])</code>         | User class for SUR estimation, both two step as well as iterated |
| <code>spreg.SURerrorGM(bigy, bigX, w[, regimes, ...])</code>  | User class for SUR Error estimation by Maximum Likelihood        |
| <code>spreg.SURerrorML(bigy, bigX, w[, regimes, ...])</code>  | User class for SUR Error estimation by Maximum Likelihood        |
| <code>spreg.SURlagIV(bigy, bigX[, bigyend, bigq, ...])</code> | User class for spatial lag estimation using IV                   |
| <code>spreg.ThreeSLS(bigy, bigX, bigyend, bigq[, ...])</code> | User class for 3SLS estimation                                   |

### 3.17.1 spreg.SUR

```
class spreg.SUR(bigy, bigX, w=None, regimes=None, nonspat_diag=True, spat_diag=False,  
                vm=False, iter=False, maxiter=5, epsilon=1e-05, verbose=False, name_bigy=None,  
                name_bigX=None, name_ds=None, name_w=None, name_regimes=None)
```

User class for SUR estimation, both two step as well as iterated

#### Parameters

**bigy** [dictionary] with vector for dependent variable by equation

**bigX** [dictionary] with matrix of explanatory variables by equation (note, already includes constant term)

**w** [spatial weights object] default = None

**regimes** [list] default = None. List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**nonspat\_diag: boolean** flag for non-spatial diagnostics, default = True

**spat\_diag** [boolean] flag for spatial diagnostics, default = False

**iter** [boolean] whether or not to use iterated estimation. default = False

**maxiter** [int] maximum iterations; default = 5

**epsilon** [float] precision criterion to end iterations. default = 0.00001

**verbose** [boolean] flag to print out iteration number and value of log det(sig) at the beginning and the end of the iteration

**name\_bigy** [dictionary] with name of dependent variable for each equation. default = None, but should be specified is done when sur\_stackxy is used

**name\_bigX** [dictionary] with names of explanatory variables for each equation. default = None, but should be specified is done when sur\_stackxy is used

**name\_ds** [string] name for the data set

**name\_w** [string] name for the weights file

**name\_regimes** [string] name of regime variable for use in the output

#### Examples

First import libpysal to load the spatial analysis tools.

```
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

The specification of the model to be estimated can be provided as lists. Each equation should be listed separately. In this example, equation 1 has HR80 as dependent variable and PS80 and UE80 as exogenous regressors. For equation 2, HR90 is the dependent variable, and PS90 and UE90 the exogenous regressors.

```
>>> y_var = ['HR80', 'HR90']  
>>> x_var = [['PS80', 'UE80'], ['PS90', 'UE90']]
```



Although not required for this method, we can load a weights matrix file to allow for spatial diagnostics.

```
>>> w = libpysal.weights.Queen.from_shapefile(libpysal.examples.get_path("NAT.shp
↳"))
>>> w.transform='r'
```

The SUR method requires data to be provided as dictionaries. PySAL provides the tool `sur_dictxy` to create these dictionaries from the list of variables. The line below will create four dictionaries containing respectively the dependent variables (`bigy`), the regressors (`bigX`), the dependent variables' names (`bigyvars`) and regressors' names (`bigXvars`). All these will be created from the database (`db`) and lists of variables (`y_var` and `x_var`) created above.

```
>>> bigy, bigX, bigyvars, bigXvars = pysal.spreg.sur_utils.sur_dictxy(db, y_var, x_var)
```

We can now run the regression and then have a summary of the output by typing: `'print(reg.summary)'`

```
>>> reg = SUR(bigy, bigX, w=w, name_bigy=bigyvars, name_bigX=bigXvars, spat_diag=True,
↳name_ds="nat")
>>> print(reg.summary)
REGRESSION
-----
SUMMARY OF OUTPUT: SEEMINGLY UNRELATED REGRESSIONS (SUR)
-----
Data set          :      nat
Weights matrix    :      unknown
Number of Equations :          2          Number of Observations:      ↳
↳3085
Log likelihood (SUR): -19902.966          Number of Iterations :      ↳
↳ 1
-----

SUMMARY OF EQUATION 1
-----
Dependent Variable :      HR80          Number of Variables :      ↳
↳ 3
Mean dependent var :      6.9276          Degrees of Freedom :      ↳
↳3082
S.D. dependent var :      6.8251
-----

↳--
Variable      Coefficient      Std.Error      z-Statistic      ↳
↳Probability
-----
↳--
Constant_1      5.1390718      0.2624673      19.5798587      0.
↳0000000
PS80      0.6776481      0.1219578      5.5564132      0.
↳0000000
UE80      0.2637240      0.0343184      7.6846277      0.
↳0000000
-----

↳--

SUMMARY OF EQUATION 2
-----
Dependent Variable :      HR90          Number of Variables :      ↳
↳ 3
```

(continues on next page)

(continued from previous page)

```

Mean dependent var   :      6.1829          Degrees of Freedom   :
↳3082
S.D. dependent var   :      6.6403

-----
↳--
Variable      Coefficient      Std.Error      z-Statistic
↳Probability
-----
↳--
Constant_2      3.6139403      0.2534996      14.2561949      0.
↳0000000
PS90            1.0260715      0.1121662      9.1477755      0.
↳0000000
UE90            0.3865499      0.0341996      11.3027760      0.
↳0000000
-----
↳--

REGRESSION DIAGNOSTICS
                        TEST      DF      VALUE      PROB
                        LM test on Sigma      1      680.168      0.0000
                        LR test on Sigma      1      768.385      0.0000

OTHER DIAGNOSTICS - CHOW TEST BETWEEN EQUATIONS
                        VARIABLES      DF      VALUE      PROB
Constant_1, Constant_2      1      26.729      0.0000
PS80, PS90      1      8.241      0.0041
UE80, UE90      1      9.384      0.0022

DIAGNOSTICS FOR SPATIAL DEPENDENCE
TEST      DF      VALUE      PROB
Lagrange Multiplier (error)      2      1333.586      0.0000
Lagrange Multiplier (lag)      2      1275.821      0.0000

ERROR CORRELATION MATRIX
EQUATION 1  EQUATION 2
1.000000    0.469548
0.469548    1.000000

===== END OF REPORT
↳=====

```

### Attributes

**bigy** [dictionary] with y values

**bigX** [dictionary] with X values

**bigXX** [dictionary] with  $X_t'X_r$  cross-products

**bigXy** [dictionary] with  $X_t'y_r$  cross-products

**n\_eq** [int] number of equations

**n** [int] number of observations in each cross-section

**bigK** [array] vector with number of explanatory variables (including constant) for each equation

**bOLS** [dictionary] with OLS regression coefficients for each equation

**olsE** [array] N x n\_eq array with OLS residuals for each equation  
**bSUR** [dictionary] with SUR regression coefficients for each equation  
**varb** [array] variance-covariance matrix  
**bigE** [array] n by n\_eq array of residuals  
**sig\_ols** [array] Sigma matrix for OLS residuals (diagonal)  
**ldetS0** [float] log det(Sigma) for null model (OLS by equation)  
**niter** [int] number of iterations (=0 for iter=False)  
**corr** [array] inter-equation error correlation matrix  
**llik** [float] log-likelihood (including the constant pi)  
**sur\_inf** [dictionary] with standard error, asymptotic t and p-value, one for each equation  
**lrtest** [tuple] Likelihood Ratio test on off-diagonal elements of sigma (tuple with test,df,p-value)  
**lmtest** [tuple] Lagrange Multiplier test on off-diagonal elements of sigma (tuple with test,df,p-value)  
**lmetest** [tuple] Lagrange Multiplier test on error spatial autocorrelation in SUR (tuple with test, df, p-value)  
**lmlagtest** [tuple] Lagrange Multiplier test on spatial lag autocorrelation in SUR (tuple with test, df, p-value)  
**surchow** [array] list with tuples for Chow test on regression coefficients. each tuple contains test value, degrees of freedom, p-value  
**name\_bigy** [dictionary] with name of dependent variable for each equation  
**name\_bigX** [dictionary] with names of explanatory variables for each equation  
**name\_ds** [string] name for the data set  
**name\_w** [string] name for the weights file  
**name\_regimes** [string] name of regime variable for use in the output

**\_\_init\_\_** (*self, bigy, bigX, w=None, regimes=None, nonspat\_diag=True, spat\_diag=False, vm=False, iter=False, maxiter=5, epsilon=1e-05, verbose=False, name\_bigy=None, name\_bigX=None, name\_ds=None, name\_w=None, name\_regimes=None*)  
 Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, bigy, bigX[, w, regimes, ...])</code> | Initialize self. |
|--|------------------|

---

### 3.17.2 spreg.SURerrorGM

**class** `spreg.SURerrorGM` (*bigy, bigX, w, regimes=None, nonspat\_diag=True, spat\_diag=False, vm=False, name\_bigy=None, name\_bigX=None, name\_ds=None, name\_w=None, name\_regimes=None*)

User class for SUR Error estimation by Maximum Likelihood

#### Parameters

**bigy** [dictionary] with vectors of dependent variable, one for each equation

**bigX** [dictionary] with matrices of explanatory variables, one for each equation

**w** [spatial weights object]

**regimes** [list] List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**nonspat\_diag** [boolean] flag for non-spatial diagnostics, default = False

**spat\_diag** [boolean] flag for spatial diagnostics, default = False (to be implemented)

**vm** [boolean] flag for asymptotic variance for lambda and Sigma, default = False (to be implemented)

**name\_bigy** [dictionary] with name of dependent variable for each equation. default = None, but should be specified is done when sur\_stackxy is used

**name\_bigX** [dictionary] with names of explanatory variables for each equation. default = None, but should be specified is done when sur\_stackxy is used

**name\_ds** [string] name for the data set

**name\_w** [string] name for the weights file

**name\_regimes** [string] name of regime variable for use in the output

## Examples

First import pysal to load the spatial analysis tools.

```
>>> import pysal
```

Open data on NCOVR US County Homicides (3085 areas) using `pysal.open()`. This is the DBF associated with the NAT shapefile. Note that `pysal.open()` also reads data in CSV format.

```
>>> db = pysal.open(pysal.examples.get_path("NAT.dbf"), 'r')
```

The specification of the model to be estimated can be provided as lists. Each equation should be listed separately. Equation 1 has HR80 as dependent variable, and PS80 and UE80 as exogenous regressors. For equation 2, HR90 is the dependent variable, and PS90 and UE90 the exogenous regressors.

```
>>> y_var = ['HR80', 'HR90']
>>> x_var = [['PS80', 'UE80'], ['PS90', 'UE90']]
>>> yend_var = [['RD80'], ['RD90']]
>>> q_var = [['FP79'], ['FP89']]
```

The SUR method requires data to be provided as dictionaries. PySAL provides the tool `sur_dictxy` to create these dictionaries from the list of variables. The line below will create four dictionaries containing respectively the dependent variables (`bigy`), the regressors (`bigX`), the dependent variables' names (`bigyvars`) and regressors' names (`bigXvars`). All these will be created from the database (`db`) and lists of variables (`y_var` and `x_var`) created above.

```
>>> bigy, bigX, bigyvars, bigXvars = pysal.spreg.sur_utils.sur_dictxy(db, y_var, x_var)
```

To run a spatial error model, we need to specify the spatial weights matrix. To do that, we can open an already existing `gal` file or create a new one. In this example, we will create a new one from `NAT.shp` and transform it to row-standardized.

```
>>> w = pysal.queen_from_shapefile(pysal.examples.get_path("NAT.shp"))
>>> w.transform='r'
```

We can now run the regression and then have a summary of the output by typing: `print(reg.summary)`

Alternatively, we can just check the betas and standard errors, asymptotic t and p-value of the parameters:

```
>>> reg = SURerrorGM(bigy,bigX,w=w,name_bigy=bigyvars,name_bigX=bigXvars,name_ds=
↳ "NAT",name_w="nat_queen")
>>> reg.bSUR
{0: array([[ 3.9774686 ],
           [ 0.8902122 ],
           [ 0.43050364]]), 1: array([[ 2.93679118],
           [ 1.11002827],
           [ 0.48761542]])}
>>> reg.sur_inf
{0: array([[ 0.37251477, 10.67734473,  0.          ],
           [ 0.14224297,  6.25839157,  0.          ],
           [ 0.04322388,  9.95985619,  0.          ]]), 1: array([[ 0.33694902,  8.
↳ 71583239,  0.          ],
           [ 0.13413626,  8.27537784,  0.          ],
           [ 0.04033105, 12.09032295,  0.          ]])}
```

### Attributes

- n** [int] number of observations in each cross-section
- n\_eq** [int] number of equations
- bigy** [dictionary] with vectors of dependent variable, one for each equation
- bigX** [dictionary] with matrices of explanatory variables, one for each equation
- bigK** [array]  $n\_eq \times 1$  array with number of explanatory variables by equation
- bigylag** [dictionary] spatially lagged dependent variable
- bigXlag** [dictionary] spatially lagged explanatory variable
- lamsur** [float] spatial autoregressive coefficient in ML SUR Error
- bSUR** [array] beta coefficients in ML SUR Error
- varb** [array] variance of beta coefficients in ML SUR Error
- sig** [array] error variance-covariance matrix in ML SUR Error
- bigE** [array]  $n$  by  $n\_eq$  matrix of vectors of residuals for each equation
- sur\_inf** [array] inference for regression coefficients, stand. error, t, p
- surchow** [array] list with tuples for Chow test on regression coefficients. each tuple contains test value, degrees of freedom, p-value
- name\_bigy** [dictionary] with name of dependent variable for each equation
- name\_bigX** [dictionary] with names of explanatory variables for each equation
- name\_ds** [string] name for the data set
- name\_w** [string] name for the weights file
- name\_regimes** [string] name of regime variable for use in the output

```
__init__(self, bigy, bigX, w, regimes=None, nonspat_diag=True, spat_diag=False,
          vm=False, name_bigy=None, name_bigX=None, name_ds=None, name_w=None,
          name_regimes=None)
    Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, bigy, bigX, w[, regimes, ...])</code> | Initialize self. |
|--|------------------|

---

### 3.17.3 spreg.SURerrorML

```
class spreg.SURerrorML(bigy, bigX, w, regimes=None, nonspat_diag=True, spat_diag=False,
                       vm=False, epsilon=1e-07, name_bigy=None, name_bigX=None,
                       name_ds=None, name_w=None, name_regimes=None)
    User class for SUR Error estimation by Maximum Likelihood
```

#### Parameters

**bigy** [dictionary] with vectors of dependent variable, one for each equation

**bigX** [dictionary] with matrices of explanatory variables, one for each equation

**w** [spatial weights object]

**regimes** [list] default = None. List of n values with the mapping of each observation to a regime. Assumed to be aligned with 'x'.

**epsilon** [float] convergence criterion for ML iterations. default 0.0000001

**nonspat\_diag** [boolean] flag for non-spatial diagnostics, default = True

**spat\_diag** [boolean] flag for spatial diagnostics, default = False

**vm** [boolean] flag for asymptotic variance for lambda and Sigma, default = False

**name\_bigy** [dictionary] with name of dependent variable for each equation. default = None, but should be specified is done when sur\_stackxy is used

**name\_bigX** [dictionary] with names of explanatory variables for each equation. default = None, but should be specified is done when sur\_stackxy is used

**name\_ds** [string] name for the data set

**name\_w** [string] name for the weights file

**name\_regimes** [string] name of regime variable for use in the output

## Examples

First import libpysal to load the spatial analysis tools.

```
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using libpysal.io.open(). This is the DBF associated with the NAT shapefile. Note that libpysal.io.open() also reads data in CSV format.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

The specification of the model to be estimated can be provided as lists. Each equation should be listed separately. Equation 1 has HR80 as dependent variable, and PS80 and UE80 as exogenous regressors. For equation 2, HR90 is the dependent variable, and PS90 and UE90 the exogenous regressors.

```
>>> y_var = ['HR80', 'HR90']
>>> x_var = [['PS80', 'UE80'], ['PS90', 'UE90']]
>>> yend_var = [['RD80'], ['RD90']]
>>> q_var = [['FP79'], ['FP89']]
```

The SUR method requires data to be provided as dictionaries. PySAL provides the tool `sur_dictxy` to create these dictionaries from the list of variables. The line below will create four dictionaries containing respectively the dependent variables (`bigy`), the regressors (`bigX`), the dependent variables' names (`bigyvars`) and regressors' names (`bigXvars`). All these will be created from the database (`db`) and lists of variables (`y_var` and `x_var`) created above.

```
>>> bigy, bigX, bigyvars, bigXvars = pysal.spreg.sur_utils.sur_dictxy(db, y_var, x_var)
```

To run a spatial error model, we need to specify the spatial weights matrix. To do that, we can open an already existing `gal` file or create a new one. In this example, we will create a new one from `NAT.shp` and transform it to row-standardized.

```
>>> w = libpysal.weights.Queen.from_shapefile(libpysal.examples.get_path("NAT.shp")
↳ ")
>>> w.transform='r'
```

We can now run the regression and then have a summary of the output by typing: `print(reg.summary)`

Alternatively, we can just check the betas and standard errors, asymptotic t and p-value of the parameters:

```
>>> reg = SURerrorML(bigy, bigX, w=w, name_bigy=bigyvars, name_bigX=bigXvars, name_ds=
↳ "NAT", name_w="nat_queen")
>>> reg.bSUR
{0: array([[ 4.0222855 ],
          [ 0.88489646],
          [ 0.42402853]]), 1: array([[ 3.04923009],
          [ 1.10972634],
          [ 0.47075682]])}
```

```
>>> reg.sur_inf
{0: array([[ 0.36692181, 10.96224141, 0.          ],
          [ 0.14129077,  6.26294579, 0.          ],
          [ 0.04267954,  9.93517021, 0.          ]]), 1: array([[ 0.33139969,  9.
↳ 20106497, 0.          ],
          [ 0.13352591,  8.31094371, 0.          ],
          [ 0.04004097, 11.756878 , 0.          ]])}
```

### Attributes

**n** [int] number of observations in each cross-section

**n2** [int]  $n/2$

**n\_eq** [int] number of equations

**bigy** [dictionary] with vectors of dependent variable, one for each equation

**bigX** [dictionary] with matrices of explanatory variables, one for each equation

**bigK** [array]  $n_{eq} \times 1$  array with number of explanatory variables by equation

**bigylag** [dictionary] spatially lagged dependent variable

**bigXlag** [dictionary] spatially lagged explanatory variable

**lamols** [array] spatial autoregressive coefficients from equation by equation ML-Error estimation

**clikerr** [float] concentrated log-likelihood from equation by equation ML-Error estimation (no constant)

**bsUR0** [array] SUR estimation for betas without spatial autocorrelation

**llik** [float] log-likelihood for classic SUR estimation (includes constant)

**lamsur** [float] spatial autoregressive coefficient in ML SUR Error

**bsUR** [array] beta coefficients in ML SUR Error

**varb** [array] variance of beta coefficients in ML SUR Error

**sig** [array] error variance-covariance matrix in ML SUR Error

**bigE** [array] n by n\_eq matrix of vectors of residuals for each equation

**cliksurerr** [float] concentrated log-likelihood from ML SUR Error (no constant)

**sur\_inf** [array] inference for regression coefficients, stand. error, t, p

**errllik** [float] log-likelihood for error model without SUR (with constant)

**surerrllik** [float] log-likelihood for SUR error model (with constant)

**lrtest** [tuple] likelihood ratio test for off-diagonal Sigma elements

**likrlambda** [tuple] likelihood ratio test on spatial autoregressive coefficients

**vm** [array] asymptotic variance matrix for lambda and Sigma (only for vm=True)

**lamsetp** [array] inference for lambda, stand. error, t, p (only for vm=True)

**lamtest** [tuple] with test for constancy of lambda across equations (test value, degrees of freedom, p-value)

**joinlam** [tuple] with test for joint significance of lambda across equations (test value, degrees of freedom, p-value)

**surchow** [list] with tuples for Chow test on regression coefficients. each tuple contains test value, degrees of freedom, p-value

**name\_bigy** [dictionary] with name of dependent variable for each equation

**name\_bigX** [dictionary] with names of explanatory variables for each equation

**name\_ds** [string] name for the data set

**name\_w** [string] name for the weights file

**name\_regimes** [string] name of regime variable for use in the output

**\_\_init\_\_** (*self, bigy, bigX, w, regimes=None, nonspat\_diag=True, spat\_diag=False, vm=False, epsilon=1e-07, name\_bigy=None, name\_bigX=None, name\_ds=None, name\_w=None, name\_regimes=None*)  
Initialize self. See help(type(self)) for accurate signature.



## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, bigy, bigX, w[, regimes, ...])</code> | Initialize self. |
|--|------------------|

---

### 3.17.4 spreg.SURLagIV

```
class spreg.SURLagIV(bigy, bigX, bigyend=None, bigq=None, w=None, regimes=None, vm=False,
    regime_lag_sep=False, w_lags=1, lag_q=True, nonspat_diag=True,
    spat_diag=False, name_bigy=None, name_bigX=None, name_bigyend=None,
    name_bigq=None, name_ds=None, name_w=None, name_regimes=None)
```

User class for spatial lag estimation using IV

#### Parameters

**bigy** [dictionary] with vector for dependent variable by equation

**bigX** [dictionary] with matrix of explanatory variables by equation (note, already includes constant term)

**bigyend** [dictionary] with matrix of endogenous variables by equation (optional)

**bigq** [dictionary] with matrix of instruments by equation (optional)

**w** [spatial weights object, required]

**vm** [boolean] listing of full variance-covariance matrix, default = False

**w\_lags** [integer] order of spatial lags for WX instruments, default = 1

**lag\_q** [boolean] flag to apply spatial lag to other instruments, default = True

**nonspat\_diag** [boolean] flag for non-spatial diagnostics, default = True

**spat\_diag** [boolean] flag for spatial diagnostics, default = False

**name\_bigy** [dictionary] with name of dependent variable for each equation. default = None, but should be specified. is done when sur\_stackxy is used.

**name\_bigX** [dictionary] with names of explanatory variables for each equation. default = None, but should be specified. is done when sur\_stackxy is used.

**name\_bigyend** [dictionary] with names of endogenous variables for each equation. default = None, but should be specified. is done when sur\_stackZ is used.

**name\_bigq** [dictionary] with names of instrumental variables for each equations. default = None, but should be specified. is done when sur\_stackZ is used.

**name\_ds** [string] name for the data set

**name\_w** [string] name for the spatial weights

#### Examples

First import libpysal to load the spatial analysis tools.

```
>>> import libpysal
```

Open data on NCOVR US County Homicides (3085 areas) using `libpysal.io.open()`. This is the DBF associated with the NAT shapefile. Note that `libpysal.io.open()` also reads data in CSV format.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("NAT.dbf"), 'r')
```

The specification of the model to be estimated can be provided as lists. Each equation should be listed separately. Although not required, in this example we will specify additional endogenous regressors. Equation 1 has HR80 as dependent variable, PS80 and UE80 as exogenous regressors, RD80 as endogenous regressor and FP79 as additional instrument. For equation 2, HR90 is the dependent variable, PS90 and UE90 the exogenous regressors, RD90 as endogenous regressor and FP99 as additional instrument

```
>>> y_var = ['HR80', 'HR90']
>>> x_var = [['PS80', 'UE80'], ['PS90', 'UE90']]
>>> yend_var = [['RD80'], ['RD90']]
>>> q_var = [['FP79'], ['FP89']]
```

The SUR method requires data to be provided as dictionaries. PySAL provides two tools to create these dictionaries from the list of variables: `sur_dictxy` and `sur_dictZ`. The tool `sur_dictxy` can be used to create the dictionaries for Y and X, and `sur_dictZ` for endogenous variables (`yend`) and additional instruments (`q`).

```
>>> bigy, bigX, bigyvars, bigXvars = pysal.spreg.sur_utils.sur_dictxy(db, y_var, x_var)
>>> bigyend, bigyendvars = pysal.spreg.sur_utils.sur_dictZ(db, yend_var)
>>> bigq, bigqvars = pysal.spreg.sur_utils.sur_dictZ(db, q_var)
```

To run a spatial lag model, we need to specify the spatial weights matrix. To do that, we can open an already existing gal file or create a new one. In this example, we will create a new one from NAT.shp and transform it to row-standardized.

```
>>> w = libpysal.weights.Queen.from_shapefile(libpysal.examples.get_path("NAT.shp
↪"))
>>> w.transform='r'
```

We can now run the regression and then have a summary of the output by typing: `print(reg.summary)`

Alternatively, we can just check the betas and standard errors, asymptotic t and p-value of the parameters:

```
>>> reg = SURlagIV(bigy, bigX, bigyend, bigq, w=w, name_bigy=bigyvars, name_
↪bigX=bigXvars, name_bigyend=bigyendvars, name_bigq=bigqvars, name_ds="NAT", name_w=
↪"nat_queen")
>>> reg.b3SLS
{0: array([[ 6.95472387],
          [ 1.44044301],
          [-0.00771893],
          [ 3.65051153],
          [ 0.00362663]]), 1: array([[ 5.61101925],
          [ 1.38716801],
          [-0.15512029],
          [ 3.1884457 ],
          [ 0.25832185]])}
```

```
>>> reg.tslls_inf
{0: array([[ 0.49128435, 14.15620899,  0.          ],
          [ 0.11516292, 12.50787151,  0.          ],
          [ 0.03204088, -0.2409087 ,  0.80962588],
          [ 0.1876025 , 19.45875745,  0.          ],
          [ 0.05450628,  0.06653605,  0.94695106]]), 1: array([[ 0.44969956, 12.
↪47726211,  0.          ],
          [ 0.10440241, 13.28674277,  0.          ],
          [ 0.04150243, -3.73761961,  0.00018577],
```

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```
[ 0.19133145, 16.66451427, 0.    ],
[ 0.04394024,  5.87893596, 0.    ]]) }
```

## Attributes

**w** [spatial weights object]

**bigy** [dictionary] with y values

**bigZ** [dictionary] with matrix of exogenous and endogenous variables for each equation

**bigyend** [dictionary] with matrix of endogenous variables for each equation; contains Wy only if no other endogenous specified

**bigq** [dictionary] with matrix of instrumental variables for each equation; contains WX only if no other endogenous specified

**bigZHZH** [dictionary] with matrix of cross products Zhat\_r'Zhat\_s

**bigZHy** [dictionary] with matrix of cross products Zhat\_r'y\_end\_s

**n\_eq** [int] number of equations

**n** [int] number of observations in each cross-section

**bigK** [array] vector with number of explanatory variables (including constant, exogenous and endogenous) for each equation

**b2SLS** [dictionary] with 2SLS regression coefficients for each equation

**tslsE** [array] N x n\_eq array with OLS residuals for each equation

**b3SLS** [dictionary] with 3SLS regression coefficients for each equation

**varb** [array] variance-covariance matrix

**sig** [array] Sigma matrix of inter-equation error covariances

**resids** [array] n by n\_eq array of residuals

**corr** [array] inter-equation 3SLS error correlation matrix

**tsls\_inf** [dictionary] with standard error, asymptotic t and p-value, one for each equation

**joinrho** [tuple] test on joint significance of spatial autoregressive coefficient. tuple with test statistic, degrees of freedom, p-value

**surchow** [array] list with tuples for Chow test on regression coefficients each tuple contains test value, degrees of freedom, p-value

**name\_w** [string] name for the spatial weights

**name\_ds** [string] name for the data set

**name\_bigy** [dictionary] with name of dependent variable for each equation

**name\_bigX** [dictionary] with names of explanatory variables for each equation

**name\_bigyend** [dictionary] with names of endogenous variables for each equation

**name\_bigq** [dictionary] with names of instrumental variables for each equations

```
__init__(self, bigy, bigX, bigyend=None, bigq=None, w=None, regimes=None, vm=False,
         regime_lag_sep=False, w_lags=1, lag_q=True, nonspat_diag=True, spat_diag=False,
         name_bigy=None, name_bigX=None, name_bigyend=None, name_bigq=None,
         name_ds=None, name_w=None, name_regimes=None)
Initialize self. See help(type(self)) for accurate signature.
```

## Methods

---

|   |                  |
|---|------------------|
| <code>__init__(self, bigy, bigX[, bigyend, bigq, ...])</code> | Initialize self. |
|---|------------------|

---

## 3.18 Diagnostics

Diagnostic tests are useful for identifying model fit, sufficiency, and specification correctness.

|   |   |
|---|---|
| <code>spreg.diagnostics.f_stat(reg)</code>                  | Calculates the f-statistic and associated p-value of the regression.                                |
| <code>spreg.diagnostics.t_stat(reg[, z_stat])</code>        | Calculates the t-statistics (or z-statistics) and associated p-values.                              |
| <code>spreg.diagnostics.r2(reg)</code>                      | Calculates the R <sup>2</sup> value for the regression.   |
| <code>spreg.diagnostics.ar2(reg)</code>                     | Calculates the adjusted R <sup>2</sup> value for the regression.                                    |
| <code>spreg.diagnostics.se_betas(reg)</code>                | Calculates the standard error of the regression coefficients.                                       |
| <code>spreg.diagnostics.log_likelihood(reg)</code>          | Calculates the log-likelihood value for the regression.   |
| <code>spreg.diagnostics.akaike(reg)</code>                  | Calculates the Akaike Information Criterion.  |
| <code>spreg.diagnostics.schwarz(reg)</code>                 | Calculates the Schwarz Information Criterion.   |
| <code>spreg.diagnostics.condition_index(reg)</code>         | Calculates the multicollinearity condition index according to Belsey, Kuh and Welsh (1980) [BKW05]. |
| <code>spreg.diagnostics.jarque_bera(reg)</code>             | Jarque-Bera test for normality in the residuals.  |
| <code>spreg.diagnostics.breusch_pagan(reg[, z])</code>      | Calculates the Breusch-Pagan test statistic to check for heteroscedasticity.                        |
| <code>spreg.diagnostics.white(reg)</code>                   | Calculates the White test to check for heteroscedasticity.  |
| <code>spreg.diagnostics.koenker_bassett(reg[, z])</code>    | Calculates the Koenker-Bassett test statistic to check for heteroscedasticity.                      |
| <code>spreg.diagnostics.vif(reg)</code>                     | Calculates the variance inflation factor for each independent variable.                             |
| <code>spreg.diagnostics.likratiotest(reg0, reg1)</code>     | Likelihood ratio test statistic [Gre03]   |
| <code>spreg.diagnostics_sp.LMtests(ols, w[, tests])</code>  | Lagrange Multiplier tests.  |
| <code>spreg.diagnostics_sp.MoranRes(ols, w[, z])</code>     | Moran's I for spatial autocorrelation in residuals from OLS regression                              |
| <code>spreg.diagnostics_sp.AKtest(iv, w[, case])</code>     | Moran's I test of spatial autocorrelation for IV estimation.  |
| <code>spreg.diagnostics_sur.sur_setp(bigB, varb)</code>     | Utility to compute standard error, t and p-value  |
| <code>spreg.diagnostics_sur.sur_lrtest(n, n_eq, ...)</code> | Likelihood Ratio test on off-diagonal elements of Sigma   |
| <code>spreg.diagnostics_sur.sur_lmtest(n, n_eq, sig)</code> | Lagrange Multiplier test on off-diagonal elements of Sigma  |

Continued on next page

Table 63 – continued from previous page

|  |  |
|--|--|
| <code>spreg.diagnostics_sur.lam_setp(lam, vm)</code>           | Standard errors, t-test and p-value for lambda in SUR Error ML   |
| <code>spreg.diagnostics_sur.surLMe(n_eq, WS, bigE, sig)</code> | Lagrange Multiplier test on error spatial autocorrelation in SUR |
| <code>spreg.diagnostics_sur.surLMlag(n_eq, WS, ...)</code>     | Lagrange Multiplier test on lag spatial autocorrelation in SUR   |

### 3.18.1 spreg.diagnostics.f\_stat

`spreg.diagnostics.f_stat(reg)`

Calculates the f-statistic and associated p-value of the regression. [Gre03]. (For two stage least squares see `f_stat_tsls`)

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**fs\_result** [tuple] includes value of F statistic and associated p-value

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the F-statistic for the regression.

```
>>> testresult = diagnostics.f_stat(reg)
```

Print the results tuple, including the statistic and its significance.

```
>>> print("%12.12f"%testresult[0], "%12.12f"%testresult[1])
('28.385629224695', '0.000000009341')
```

### 3.18.2 spreg.diagnostics.t\_stat

`spreg.diagnostics.t_stat` (*reg*, *z\_stat=False*)

Calculates the t-statistics (or z-statistics) and associated p-values. [Gre03]

#### Parameters

**reg** [regression object] output instance from a regression model

**z\_stat** [boolean] If True run z-stat instead of t-stat

#### Returns

**ts\_result** [list of tuples] each tuple includes value of t statistic (or z statistic) and associated p-value

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.open(libpysal.examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate t-statistics for the regression coefficients.

```
>>> testresult = diagnostics.t_stat(reg)
```

Print the tuples that contain the t-statistics and their significances.

```
>>> print("%12.12f"%testresult[0][0], "%12.12f"%testresult[0][1], "%12.12f"%
↳testresult[1][0], "%12.12f"%testresult[1][1], "%12.12f"%testresult[2][0], "%12.
↳12f"%testresult[2][1])
('14.490373143689', '0.000000000000', '-4.780496191297', '0.000018289595', '-2.
↳654408642718', '0.010874504910')
```

### 3.18.3 spreg.diagnostics.r2

`spreg.diagnostics.r2` (*reg*)

Calculates the  $R^2$  value for the regression. [Gre03]

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**r2\_result** [float] value of the coefficient of determination for the regression

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the  $R^2$  value for the regression.

```
>>> testresult = diagnostics.r2(reg)
```

Print the result.

```
>>> print("%1.8f"%testresult)
0.55240404
```

### 3.18.4 spreg.diagnostics.ar2

`spreg.diagnostics.ar2(reg)`

Calculates the adjusted  $R^2$  value for the regression. [Gre03]

**Parameters**

**reg** [regression object] output instance from a regression model

**Returns**

**ar2\_result** [float] value of  $R^2$  adjusted for the number of explanatory variables.

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the adjusted  $R^2$  value for the regression. `>>> testresult = diagnostics.ar2(reg)`

Print the result.

```
>>> print("%1.8f"%testresult)
0.53294335
```

### 3.18.5 spreg.diagnostics.se\_betas

`spreg.diagnostics.se_betas(reg)`

Calculates the standard error of the regression coefficients. [Gre03]

**Parameters**

**reg** [regression object] output instance from a regression model

**Returns**



**se\_result** [array] includes standard errors of each coefficient (1 x k)

### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the standard errors of the regression coefficients.

```
>>> testresult = diagnostics.se_betas(reg)
```

Print the vector of standard errors.

```
>>> testresult
array([ 4.73548613,  0.33413076,  0.10319868])
```

## 3.18.6 spreg.diagnostics.log\_likelihood

`spreg.diagnostics.log_likelihood(reg)`

Calculates the log-likelihood value for the regression. [Gre03]

### Parameters

**reg** [regression object] output instance from a regression model

### Returns

**ll\_result** [float] value for the log-likelihood of the regression.

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the log-likelihood for the regression.

```
>>> testresult = diagnostics.log_likelihood(reg)
```

Print the result.

```
>>> testresult
-187.3772388121491
```

### 3.18.7 spreg.diagnostics.akaik

`spreg.diagnostics.akaik`(*reg*)

Calculates the Akaike Information Criterion. [Aka74]

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**aic\_result** [scalar] value for Akaike Information Criterion of the regression.

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
```

(continues on next page)

(continued from previous page)

```
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = y.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the Akaike Information Criterion (AIC).

```
>>> testresult = diagnostics.akaike(reg)
```

Print the result.

```
>>> testresult
380.7544776242982
```

### 3.18.8 spreg.diagnostics.schwarz

`spreg.diagnostics.schwarz` (*reg*)

Calculates the Schwarz Information Criterion. [S+78]

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**bic\_result** [scalar] value for Schwarz (Bayesian) Information Criterion of the regression.

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the Schwarz Information Criterion.

```
>>> testresult = diagnostics.schwarz(reg)
```

Print the results.

```
>>> testresult
386.42993851863008
```

### 3.18.9 spreg.diagnostics.condition\_index

`spreg.diagnostics.condition_index(reg)`

Calculates the multicollinearity condition index according to Belsey, Kuh and Welsh (1980) [BKW05].

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**ci\_result** [float] scalar value for the multicollinearity condition index.

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y, X)
```

Calculate the condition index to check for multicollinearity.

```
>>> testresult = diagnostics.condition_index(reg)
```

Print the result.

```
>>> print("%1.3f"%testresult)
6.542
```

### 3.18.10 spreg.diagnostics.jarque\_bera

`spreg.diagnostics.jarque_bera(reg)`

Jarque-Bera test for normality in the residuals. [JB80]

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**jb\_result** [dictionary] contains the statistic (jb) for the Jarque-Bera test and the associated p-value (p-value)

**df** [integer] degrees of freedom for the test (always 2)

**jb** [float] value of the test statistic

**pvalue** [float] p-value associated with the statistic (chi<sup>2</sup> distributed with 2 df)

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y, X)
```

Calculate the Jarque-Bera test for normality of residuals.

```
>>> testresult = diagnostics.jarque_bera(reg)
```

Print the degrees of freedom for the test.

```
>>> testresult['df']
2
```

Print the test statistic.

```
>>> print("%1.3f"%testresult['jb'])
1.836
```

Print the associated p-value.

```
>>> print("%1.4f"%testresult['pvalue'])
0.3994
```

### 3.18.11 spreg.diagnostics.breusch\_pagan

`spreg.diagnostics.breusch_pagan` (*reg*, *z=None*)

Calculates the Breusch-Pagan test statistic to check for heteroscedasticity. [BP79]

#### Parameters

- reg** [regression object] output instance from a regression model
- z** [array] optional input for specifying an alternative set of variables (Z) to explain the observed variance. By default this is a matrix of the squared explanatory variables ( $X^{**2}$ ) with a constant added to the first column if not already present. In the default case, the explanatory variables are squared to eliminate negative values.

#### Returns

- bp\_result** [dictionary] contains the statistic (bp) for the test and the associated p-value (p-value)
- bp** [float] scalar value for the Breusch-Pagan test statistic
- df** [integer] degrees of freedom associated with the test (k)
- pvalue** [float] p-value associated with the statistic ( $\chi^2$  distributed with k df)

#### Notes

x attribute in the reg object must have a constant term included. This is standard for spreg.OLS so no testing done to confirm constant.

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the Breusch-Pagan test for heteroscedasticity.

```
>>> testresult = diagnostics.breusch_pagan(reg)
```

Print the degrees of freedom for the test.

```
>>> testresult['df']
2
```

Print the test statistic.

```
>>> print("%1.3f"%testresult['bp'])
7.900
```

Print the associated p-value.

```
>>> print("%1.4f"%testresult['pvalue'])
0.0193
```

### 3.18.12 spreg.diagnostics.white

`spreg.diagnostics.white`(*reg*)

Calculates the White test to check for heteroscedasticity. [Whi80]

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**white\_result** [dictionary] contains the statistic (white), degrees of freedom (df) and the associated p-value (pvalue) for the White test.

**white** [float] scalar value for the White test statistic.

**df** [integer] degrees of freedom associated with the test

**pvalue** [float] p-value associated with the statistic ( $\chi^2$  distributed with k df)

## Notes

x attribute in the reg object must have a constant term included. This is standard for spreg.OLS so no testing done to confirm constant.

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the White test for heteroscedasticity.

```
>>> testresult = diagnostics.white(reg)
```

Print the degrees of freedom for the test.

```
>>> print testresult['df']
5
```

Print the test statistic.

```
>>> print ("%1.3f"%testresult['wh'])
19.946
```

Print the associated p-value.



```
>>> print("%1.4f"%testresult['pvalue'])
0.0013
```

### 3.18.13 spreg.diagnostics.koenker\_bassett

`spreg.diagnostics.koenker_bassett` (*reg*, *z=None*)

Calculates the Koenker-Bassett test statistic to check for heteroscedasticity. [KBJ82][Gre03]

#### Parameters

- reg** [regression output] output from an instance of a regression class
- z** [array] optional input for specifying an alternative set of variables (Z) to explain the observed variance. By default this is a matrix of the squared explanatory variables ( $X^{**2}$ ) with a constant added to the first column if not already present. In the default case, the explanatory variables are squared to eliminate negative values.

#### Returns

- kb\_result** [dictionary] contains the statistic (kb), degrees of freedom (df) and the associated p-value (pvalue) for the test.
- kb** [float] scalar value for the Koenker-Bassett test statistic.
- df** [integer] degrees of freedom associated with the test
- pvalue** [float] p-value associated with the statistic ( $\chi^2$  distributed)

#### Notes

x attribute in the reg object must have a constant term included. This is standard for `spreg.OLS` so no testing done to confirm constant.

#### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import diagnostics
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y,X)
```

Calculate the Koenker-Bassett test for heteroscedasticity.

```
>>> testresult = diagnostics.koenker_bassett(reg)
```

Print the degrees of freedom for the test.

```
>>> testresult['df']  
2
```

Print the test statistic.

```
>>> print("%1.3f"%testresult['kb'])  
5.694
```

Print the associated p-value.

```
>>> print("%1.4f"%testresult['pvalue'])  
0.0580
```

### 3.18.14 spreg.diagnostics.vif

`spreg.diagnostics.vif`(*reg*)

Calculates the variance inflation factor for each independent variable. For the ease of indexing the results, the constant is currently included. This should be omitted when reporting the results to the output text. [Gre03]

#### Parameters

**reg** [regression object] output instance from a regression model

#### Returns

**vif\_result** [list of tuples] each tuple includes the vif and the tolerance, the order of the variables corresponds to their order in the `reg.x` matrix

#### Examples

```
>>> import numpy as np  
>>> import libpysal  
>>> from libpysal import examples  
>>> import diagnostics  
>>> from ols import OLS
```

Read the DBF associated with the Columbus data.

```
>>> db = libpysal.io.open(examples.get_path("columbus.dbf"), "r")
```

Create the dependent variable vector.

```
>>> y = np.array(db.by_col("CRIME"))  
>>> y = np.reshape(y, (49,1))
```

Create the matrix of independent variables.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X.append(db.by_col("HOVAL"))
>>> X = np.array(X).T
```

Run an OLS regression.

```
>>> reg = OLS(y, X)
```

Calculate the variance inflation factor (VIF). >>> testresult = diagnostics.vif(reg)

Select the tuple for the income variable.

```
>>> incvif = testresult[1]
```

Print the VIF for income.

```
>>> print("%12.12f"%incvif[0])
1.333117497189
```

Print the tolerance for income.

```
>>> print("%12.12f"%incvif[1])
0.750121427487
```

Repeat for the home value variable.

```
>>> hovalvif = testresult[2]
>>> print("%12.12f"%hovalvif[0])
1.333117497189
>>> print("%12.12f"%hovalvif[1])
0.750121427487
```

### 3.18.15 spreg.diagnostics.likratiotest

`spreg.diagnostics.likratiotest` (*reg0*, *reg1*)  
Likelihood ratio test statistic [Gre03]

#### Parameters

- reg0** [regression object] for constrained model (H0)
- reg1** [regression object] for unconstrained model (H1)

#### Returns

- likratio** [dictionary] contains the statistic (*likr*), the degrees of freedom (*df*) and the p-value (*pvalue*)
- likr** [float] likelihood ratio statistic
- df** [integer] degrees of freedom
- p-value** [float] p-value

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from libpysal import examples
>>> import scipy.stats as stats
>>> import spreg.ml_lag as lag
```

Use the baltim sample data set

```
>>> db = libpysal.io.open(examples.get_path("baltim.dbf"), 'r')
>>> y_name = "PRICE"
>>> y = np.array(db.by_col(y_name)).T
>>> y.shape = (len(y), 1)
>>> x_names = ["NROOM", "NBATH", "PATIO", "FIREPL", "AC", "GAR", "AGE", "LOTSZ", "SQFT"]
>>> x = np.array([db.by_col(var) for var in x_names]).T
>>> ww = ps.open(ps.examples.get_path("baltim_q.gal"))
>>> w = ww.read()
>>> ww.close()
>>> w.transform = 'r'
```

OLS regression

```
>>> ols1 = ps.spreg.OLS(y, x)
```

ML Lag regression

```
>>> mllag1 = lag.ML_Lag(y, x, w)
```

```
>>> lr = likratiotest(ols1, mllag1)
```

```
>>> print "Likelihood Ratio Test: {0:.4f}          df: {1}          p-value: {2:.4f}".
↪format(lr["likr"], lr["df"], lr["p-value"])
Likelihood Ratio Test: 44.5721          df: 1          p-value: 0.0000
```

### 3.18.16 spreg.diagnostics\_sp.LMtests

**class** spreg.diagnostics\_sp.**LMtests** (*ols, w, tests=['all']*)

Lagrange Multiplier tests. Implemented as presented in [ABFY96]

#### Parameters

**lme** [tuple] (Only if 'lme' or 'all' was in tests). Pair of statistic and p-value for the LM error test.

**lml** [tuple] (Only if 'lml' or 'all' was in tests). Pair of statistic and p-value for the LM lag test.

**rlme** [tuple] (Only if 'rlme' or 'all' was in tests). Pair of statistic and p-value for the Robust LM error test.

**rlml** [tuple] (Only if 'rlml' or 'all' was in tests). Pair of statistic and p-value for the Robust LM lag test.

**sarma** [tuple] (Only if 'rlml' or 'all' was in tests). Pair of statistic and p-value for the SARMA test.

## Examples

```
>>> import numpy as np
>>> import libpysal
>>> from ols import OLS
```

Open the csv file to access the data for analysis

```
>>> csv = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Pull out from the csv the files we need ('HOVAL' as dependent as well as 'INC' and 'CRIME' as independent) and directly transform them into nx1 and nx2 arrays, respectively

```
>>> y = np.array([csv.by_col('HOVAL')]).T
>>> x = np.array([csv.by_col('INC'), csv.by_col('CRIME')]).T
```

Create the weights object from existing .gal file

```
>>> w = libpysal.io.open(libpysal.examples.get_path('columbus.gal'), 'r').read()
```

Row-standardize the weight object (not required although desirable in some cases)

```
>>> w.transform='r'
```

Run an OLS regression

```
>>> ols = OLS(y, x)
```

Run all the LM tests in the residuals. These diagnostics test for the presence of remaining spatial autocorrelation in the residuals of an OLS model and give indication about the type of spatial model. There are five types: presence of a spatial lag model (simple and robust version), presence of a spatial error model (simple and robust version) and joint presence of both a spatial lag as well as a spatial error model.

```
>>> lms = spreg.diagnostics_sp.LMtests(ols, w)
```

LM error test:

```
>>> print round(lms.lme[0],4), round(lms.lme[1],4)
3.0971 0.0784
```

LM lag test:

```
>>> print round(lms.lml[0],4), round(lms.lml[1],4)
0.9816 0.3218
```

Robust LM error test:

```
>>> print round(lms.rlme[0],4), round(lms.rlme[1],4)
3.2092 0.0732
```

Robust LM lag test:

```
>>> print round(lms.rlml[0],4), round(lms.rlml[1],4)
1.0936 0.2957
```

LM SARMA test:

```
>>> print round(lms.sarma[0],4), round(lms.sarma[1],4)
4.1907 0.123
```

### Attributes

**ols** [OLS] OLS regression object

**w** [W] Spatial weights instance

**tests** [list] Lists of strings with the tests desired to be performed. Values may be:

- ‘all’: runs all the options (default)
- ‘lme’: LM error test
- ‘rlme’: Robust LM error test
- ‘lml’: LM lag test
- ‘rlml’: Robust LM lag test

`__init__(self, ols, w, tests=['all'])`

Initialize self. See help(type(self)) for accurate signature.

### Methods

---

`__init__(self, ols, w[, tests])`

Initialize self.

---

## 3.18.17 spreg.diagnostics\_sp.MoranRes

**class** spreg.diagnostics\_sp.**MoranRes** (ols, w, z=False)

Moran’s I for spatial autocorrelation in residuals from OLS regression

### Parameters

**ols** [OLS] OLS regression object

**w** [W] Spatial weights instance

**z** [boolean] If set to True computes attributes eI, vI and zI. Due to computational burden of vI, defaults to False.

### Examples

```
>>> import numpy as np
>>> import libpysal
>>> from ols import OLS
```

Open the csv file to access the data for analysis

```
>>> csv = libpysal.io.open(libpysal.examples.get_path('columbus.dbf'), 'r')
```

Pull out from the csv the files we need (‘HOVAL’ as dependent as well as ‘INC’ and ‘CRIME’ as independent) and directly transform them into nx1 and nx2 arrays, respectively

```
>>> y = np.array([csv.by_col('HOVAL')]).T
>>> x = np.array([csv.by_col('INC'), csv.by_col('CRIME')]).T
```

Create the weights object from existing .gal file

```
>>> w = libpysal.io.open(libpysal.examples.get_path('columbus.gal'), 'r').read()
```

Row-standardize the weight object (not required although desirable in some cases)

```
>>> w.transform='r'
```

Run an OLS regression

```
>>> ols = OLS(y, x)
```

Run Moran's I test for residual spatial autocorrelation in an OLS model. This computes the traditional statistic applying a correction in the expectation and variance to account for the fact it comes from residuals instead of an independent variable

```
>>> m = spreg.diagnostics_sp.MoranRes(ols, w, z=True)
```

Value of the Moran's I statistic:

```
>>> print round(m.I, 4)
0.1713
```

Value of the Moran's I expectation:

```
>>> print round(m.eI, 4)
-0.0345
```

Value of the Moran's I variance:

```
>>> print round(m.vI, 4)
0.0081
```

Value of the Moran's I standardized value. This is distributed as a standard Normal(0, 1)

```
>>> print round(m.zI, 4)
2.2827
```

P-value of the standardized Moran's I value (z):

```
>>> print round(m.p_norm, 4)
0.0224
```

### Attributes

**I** [float] Moran's I statistic

**eI** [float] Moran's I expectation

**vI** [float] Moran's I variance

**zI** [float] Moran's I standardized value

**\_\_init\_\_** (*self*, *ols*, *w*, *z=False*)

Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <code>__init__(self, ols, w[, z])</code> | Initialize self. |
|--|------------------|

---

### 3.18.18 spreg.diagnostics\_sp.AKtest

**class** spreg.diagnostics\_sp.**AKtest** (*iv, w, case='nosp'*)

Moran's I test of spatial autocorrelation for IV estimation. Implemented following the original reference [AK97]

#### Parameters

- iv** [TSLS] Regression object from TSLS class
- w** [W] Spatial weights instance
- case** [string] Flag for special cases (default to 'nosp'):
  - 'nosp': Only NO spatial end. reg.
  - 'gen': General case (spatial lag + end. reg.)

#### Examples

We first need to import the needed modules. Numpy is needed to convert the data we read into arrays that spreg understands and pysal to perform all the analysis. The TSLS is required to run the model on which we will perform the tests.

```
>>> import numpy as np
>>> import libpysal
>>> from twosls import TSLS
>>> from twosls_sp import GM_Lag
```

Open data on Columbus neighborhood crime (49 areas) using libpysal.io.open(). This is the DBF associated with the Columbus shapefile. Note that libpysal.io.open() also reads data in CSV format; since the actual class requires data to be passed in as numpy arrays, the user can read their data in using any method.

```
>>> db = libpysal.io.open(libpysal.examples.get_path("columbus.dbf"), 'r')
```

Before being able to apply the diagnostics, we have to run a model and, for that, we need the input variables. Extract the CRIME column (crime rates) from the DBF file and make it the dependent variable for the regression. Note that PySAL requires this to be a numpy array of shape (n, 1) as opposed to the also common shape of (n, ) that other packages accept.

```
>>> y = np.array(db.by_col("CRIME"))
>>> y = np.reshape(y, (49,1))
```

Extract INC (income) vector from the DBF to be used as independent variables in the regression. Note that PySAL requires this to be an nxj numpy array, where j is the number of independent variables (not including a constant). By default this model adds a vector of ones to the independent variables passed in, but this can be overridden by passing constant=False.

```
>>> X = []
>>> X.append(db.by_col("INC"))
>>> X = np.array(X).T
```

In this case, we consider HOVAL (home value) as an endogenous regressor, so we acknowledge that by reading it in a different category.



```
>>> yd = []
>>> yd.append(db.by_col("HOVAL"))
>>> yd = np.array(yd).T
```

In order to properly account for the endogeneity, we have to pass in the instruments. Let us consider DISCBD (distance to the CBD) is a good one:

```
>>> q = []
>>> q.append(db.by_col("DISCBD"))
>>> q = np.array(q).T
```

Now we are good to run the model. It is an easy one line task.

```
>>> reg = TSLS(y, X, yd, q=q)
```

Now we are concerned with whether our non-spatial model presents spatial autocorrelation in the residuals. To assess this possibility, we can run the Anselin-Kelejian test, which is a version of the classical LM error test adapted for the case of residuals from an instrumental variables (IV) regression. First we need an extra object, the weights matrix, which includes the spatial configuration of the observations into the error component of the model. To do that, we can open an already existing gal file or create a new one. In this case, we will create one from `columbus.shp`.

```
>>> w = libpysal.weights.Rook.from_shapefile(libpysal.examples.get_path("columbus.
↪shp"))
```

Unless there is a good reason not to do it, the weights have to be row-standardized so every row of the matrix sums to one. Among other things, this allows to interpret the spatial lag of a variable as the average value of the neighboring observations. In PySAL, this can be easily performed in the following way:

```
>>> w.transform = 'r'
```

We are good to run the test. It is a very simple task:

```
>>> ak = AKtest(reg, w)
```

And explore the information obtained:

```
>>> print('AK test: %f          P-value: %f'%(ak.ak, ak.p))
AK test: 4.642895          P-value: 0.031182
```

The test also accomodates the case when the residuals come from an IV regression that includes a spatial lag of the dependent variable. The only requirement needed is to modify the `case` parameter when we call `AKtest`. First, let us run a spatial lag model:

```
>>> reg_lag = GM_Lag(y, X, yd, q=q, w=w)
```

And now we can run the AK test and obtain similar information as in the non-spatial model.

```
>>> ak_sp = AKtest(reg, w, case='gen')
>>> print('AK test: %f          P-value: %f'%(ak_sp.ak, ak_sp.p))
AK test: 1.157593          P-value: 0.281965
```

### Attributes

**mi** [float] Moran's I statistic for IV residuals

**ak** [float] Square of corrected Moran's I for residuals  $ak = \frac{NimesI^*}{\phi^2}$ . Note: if case='nosp' then it simplifies to the LMerror

**p** [float] P-value of the test

**\_\_init\_\_** (*self*, *iv*, *w*, *case*='nosp')

Initialize self. See help(type(self)) for accurate signature.

## Methods

---

|  |                  |
|--|------------------|
| <b>__init__</b> ( <i>self</i> , <i>iv</i> , <i>w</i> [, <i>case</i> ]) | Initialize self. |
|--|------------------|

---

### 3.18.19 spreg.diagnostics\_sur.sur\_setp

`spreg.diagnostics_sur.sur_setp` (*bigB*, *varb*)

Utility to compute standard error, t and p-value

#### Parameters

**bigB** [dictionary] of regression coefficient estimates, one vector by equation

**varb** [array] variance-covariance matrix of coefficients

#### Returns

**surinfdict** [dictionary] with standard error, t-value, and p-value array, one for each equation

### 3.18.20 spreg.diagnostics\_sur.sur\_lrtest

`spreg.diagnostics_sur.sur_lrtest` (*n*, *n\_eq*, *ldetS0*, *ldetS1*)

Likelihood Ratio test on off-diagonal elements of Sigma

#### Parameters

**n** [int] cross-sectional dimension (number of observations for an equation)

**n\_eq** [int] number of equations

**ldetS0** [float] log determinant of Sigma for OLS case

**ldetS1** [float] log determinant of Sigma for SUR case (should be iterated)

#### Returns

**(lrtest,M,pvalue)** [tuple] with value of test statistic (lrtest), degrees of freedom (M, as an integer) p-value

### 3.18.21 spreg.diagnostics\_sur.sur\_lmtest

`spreg.diagnostics_sur.sur_lmtest` (*n*, *n\_eq*, *sig*)

Lagrange Multiplier test on off-diagonal elements of Sigma

#### Parameters

**n** [int] cross-sectional dimension (number of observations for an equation)

**n\_eq** [int] number of equations

**sig** [array] inter-equation covariance matrix for null model (OLS)

#### Returns

(**lmtest**,**M**,**pvalue**) [tuple] with value of test statistic (lmtest), degrees of freedom (M, as an integer) p-value

### 3.18.22 spreg.diagnostics\_sur.lam\_setp

`spreg.diagnostics_sur.lam_setp(lam, vm)`

Standard errors, t-test and p-value for lambda in SUR Error ML

#### Parameters

**lam** [array]  $n_{eq} \times 1$  array with ML estimates for spatial error autoregressive coefficient

**vm** [array]  $n_{eq} \times n_{eq}$  subset of variance-covariance matrix for lambda and Sigma in SUR Error ML (needs to be subset from full vm)

#### Returns

: **tuple** with arrays for standard error, t-value and p-value (each element in the tuple is an  $n_{eq} \times 1$  array)

### 3.18.23 spreg.diagnostics\_sur.surLMe

`spreg.diagnostics_sur.surLMe(n_eq, WS, bigE, sig)`

Lagrange Multiplier test on error spatial autocorrelation in SUR

#### Parameters

**n\_eq** [int] number of equations

**WS** [array] spatial weights matrix in sparse form

**bigE** [array]  $n \times n_{eq}$  matrix of residuals by equation

**sig** [array] cross-equation error covariance matrix

#### Returns

(**LMe**,**n\_eq**,**pvalue**) [tuple] with value of statistic (LMe), degrees of freedom (n\_eq) and p-value

### 3.18.24 spreg.diagnostics\_sur.surLMlag

`spreg.diagnostics_sur.surLMlag(n_eq, WS, bigy, bigX, bigE, bigYP, sig, varb)`

Lagrange Multiplier test on lag spatial autocorrelation in SUR

#### Parameters

**n\_eq** [int] number of equations

**WS** [array] spatial weights matrix in sparse form

**bigy** [dictionary] with y values

**bigX** [dictionary] with X values

**bigE** [array]  $n \times n_{eq}$  matrix of residuals by equation

**bigYP** [array]  $n \times n_{eq}$  matrix of predicted values by equation

**sig** [array] cross-equation error covariance matrix

**varb** [array] variance-covariance matrix for b coefficients (inverse of Ibb)

**Returns**

**(LMlag,n\_eq,pvalue)** [tuple] with value of statistic (LMlag), degrees of freedom (n\_eq) and p-value

**REFERENCES**



## BIBLIOGRAPHY

- [Aka74] Hirotugu Akaike. A new look at the statistical model identification. *IEEE transactions on automatic control*, 19(6):716–723, 1974.
- [Ans88] Luc Anselin. *Spatial Econometrics: Methods and Models*. Kluwer, Dordrecht, 1988.
- [Ans11] Luc Anselin. GMM estimation of spatial error autocorrelation with and without heteroskedasticity. Technical Report, GeoDa Center for Geospatial Analysis and Computation, 2011.
- [ABFY96] Luc Anselin, Anil K Bera, Raymond Florax, and Mann J Yoon. Simple diagnostic tests for spatial dependence. *Regional science and urban economics*, 26(1):77–104, 1996.
- [AK97] Luc Anselin and Harry H Kelejian. Testing for spatial error autocorrelation in the presence of endogenous regressors. *International Regional Science Review*, 20(1-2):153–182, 1997.
- [ADKP10] Irani Arraiz, David M. Drukker, Harry H. Kelejian, and Ingmar R. Prucha. A spatial Cliff-Ord-type model with heteroskedastic innovations: Small and large sample results. *Journal of Regional Science*, 50(2):592–614, 2010. doi:10.1111/j.1467-9787.2009.00618.x.
- [BKW05] David A Belsley, Edwin Kuh, and Roy E Welsch. *Regression diagnostics: Identifying influential data and sources of collinearity*. Volume 571. John Wiley & Sons, 2005.
- [BP79] Trevor S Breusch and Adrian R Pagan. A simple test for heteroscedasticity and random coefficient variation. *Econometrica: Journal of the Econometric Society*, pages 1287–1294, 1979.
- [DEP13] David M Drukker, Peter Egger, and Ingmar R Prucha. On two-step estimation of a spatial autoregressive model with autoregressive disturbances and endogenous regressors. *Econometric Reviews*, 32(5-6):686–733, 2013.
- [DPR13] David M. Drukker, Ingmar R. Prucha, and Rafal Raciborski. A command for estimating spatial-autoregressive models with spatial-autoregressive disturbances and additional endogenous variables. *The Stata Journal*, 13(2):287–301, 2013. URL: <https://journals.sagepub.com/doi/abs/10.1177/1536867X1301300203>.
- [Gre03] William H Greene. *Econometric analysis*. Pearson Education India, 2003.
- [JB80] Carlos M Jarque and Anil K Bera. Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics letters*, 6(3):255–259, 1980.
- [KP99] H H Kelejian and I R Prucha. A generalized moments estimator for the autoregressive parameter in a spatial model. *Int. Econ. Rev.*, 40:509–534, 1999.
- [KP98] Harry H Kelejian and Ingmar R Prucha. A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbances. *J. Real Estate Fin. Econ.*, 17(1):99–121, 1998.
- [KBJ82] Roger Koenker and Gilbert Bassett Jr. Robust tests for heteroscedasticity based on regression quantiles. *Econometrica: Journal of the Econometric Society*, pages 43–61, 1982.

- [S+78] Gideon Schwarz and others. Estimating the dimension of a model. *The annals of statistics*, 6(2):461–464, 1978.
- [Whi80] Halbert White. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica: Journal of the Econometric Society*, pages 817–838, 1980.



## Symbols

`__init__()` (*spreg.GM\_Combo* method), 36  
`__init__()` (*spreg.GM\_Combo\_Het* method), 40  
`__init__()` (*spreg.GM\_Combo\_Het\_Regimes* method), 109  
`__init__()` (*spreg.GM\_Combo\_Hom* method), 44  
`__init__()` (*spreg.GM\_Combo\_Hom\_Regimes* method), 103  
`__init__()` (*spreg.GM\_Combo\_Regimes* method), 98  
`__init__()` (*spreg.GM\_Endog\_Error* method), 47  
`__init__()` (*spreg.GM\_Endog\_Error\_Het* method), 50  
`__init__()` (*spreg.GM\_Endog\_Error\_Het\_Regimes* method), 123  
`__init__()` (*spreg.GM\_Endog\_Error\_Hom* method), 54  
`__init__()` (*spreg.GM\_Endog\_Error\_Hom\_Regimes* method), 118  
`__init__()` (*spreg.GM\_Endog\_Error\_Regimes* method), 113  
`__init__()` (*spreg.GM\_Error* method), 27  
`__init__()` (*spreg.GM\_Error\_Het* method), 30  
`__init__()` (*spreg.GM\_Error\_Het\_Regimes* method), 88  
`__init__()` (*spreg.GM\_Error\_Hom* method), 33  
`__init__()` (*spreg.GM\_Error\_Hom\_Regimes* method), 93  
`__init__()` (*spreg.GM\_Error\_Regimes* method), 84  
`__init__()` (*spreg.GM\_Lag* method), 24  
`__init__()` (*spreg.GM\_Lag\_Regimes* method), 80  
`__init__()` (*spreg.ML\_Error* method), 20  
`__init__()` (*spreg.ML\_Error\_Regimes* method), 74  
`__init__()` (*spreg.ML\_Lag* method), 17  
`__init__()` (*spreg.ML\_Lag\_Regimes* method), 70  
`__init__()` (*spreg.OLS* method), 12  
`__init__()` (*spreg.OLS\_Regimes* method), 65  
`__init__()` (*spreg.SUR* method), 127  
`__init__()` (*spreg.SURerrorGM* method), 129  
`__init__()` (*spreg.SURerrorML* method), 132  
`__init__()` (*spreg.SURLagIV* method), 135  
`__init__()` (*spreg.TSLS* method), 57  
`__init__()` (*spreg.ThreeSLS* method), 60

`__init__()` (*spreg.diagnostics\_sp.AKtest* method), 158  
`__init__()` (*spreg.diagnostics\_sp.LMtests* method), 154  
`__init__()` (*spreg.diagnostics\_sp.MoranRes* method), 155

## A

`akaike()` (in module *spreg.diagnostics*), 142  
`AKtest` (class in *spreg.diagnostics\_sp*), 156  
`ar2()` (in module *spreg.diagnostics*), 140

## B

`breusch_pagan()` (in module *spreg.diagnostics*), 146

## C

`condition_index()` (in module *spreg.diagnostics*), 144

## F

`f_stat()` (in module *spreg.diagnostics*), 137

## G

`GM_Combo` (class in *spreg*), 33  
`GM_Combo_Het` (class in *spreg*), 37  
`GM_Combo_Het_Regimes` (class in *spreg*), 104  
`GM_Combo_Hom` (class in *spreg*), 41  
`GM_Combo_Hom_Regimes` (class in *spreg*), 98  
`GM_Combo_Regimes` (class in *spreg*), 93  
`GM_Endog_Error` (class in *spreg*), 44  
`GM_Endog_Error_Het` (class in *spreg*), 47  
`GM_Endog_Error_Het_Regimes` (class in *spreg*), 118  
`GM_Endog_Error_Hom` (class in *spreg*), 51  
`GM_Endog_Error_Hom_Regimes` (class in *spreg*), 114  
`GM_Endog_Error_Regimes` (class in *spreg*), 109  
`GM_Error` (class in *spreg*), 25  
`GM_Error_Het` (class in *spreg*), 27  
`GM_Error_Het_Regimes` (class in *spreg*), 85  
`GM_Error_Hom` (class in *spreg*), 30  
`GM_Error_Hom_Regimes` (class in *spreg*), 89

GM\_Error\_Regimes (*class in spreg*), 81

GM\_Lag (*class in spreg*), 20

GM\_Lag\_Regimes (*class in spreg*), 75

## J

jarque\_bera() (*in module spreg.diagnostics*), 145

## K

koenker\_basnett() (*in module spreg.diagnostics*),  
149

## L

lam\_setp() (*in module spreg.diagnostics\_sur*), 159

likratiotest() (*in module spreg.diagnostics*), 151

LMtests (*class in spreg.diagnostics\_sp*), 152

log\_likelihood() (*in module spreg.diagnostics*),  
141

## M

ML\_Error (*class in spreg*), 17

ML\_Error\_Regimes (*class in spreg*), 70

ML\_Lag (*class in spreg*), 13

ML\_Lag\_Regimes (*class in spreg*), 66

MoranRes (*class in spreg.diagnostics\_sp*), 154

## O

OLS (*class in spreg*), 8

OLS\_Regimes (*class in spreg*), 61

## R

r2() (*in module spreg.diagnostics*), 139

## S

schwarz() (*in module spreg.diagnostics*), 143

se\_betas() (*in module spreg.diagnostics*), 140

SUR (*class in spreg*), 124

sur\_lmtest() (*in module spreg.diagnostics\_sur*), 158

sur\_lrtest() (*in module spreg.diagnostics\_sur*), 158

sur\_setp() (*in module spreg.diagnostics\_sur*), 158

SURerrorGM (*class in spreg*), 127

SURerrorML (*class in spreg*), 130

SURlagIV (*class in spreg*), 133

surLMe() (*in module spreg.diagnostics\_sur*), 159

surLMlag() (*in module spreg.diagnostics\_sur*), 159

## T

t\_stat() (*in module spreg.diagnostics*), 138

ThreeSLS (*class in spreg*), 58

TSLS (*class in spreg*), 54

## V

vif() (*in module spreg.diagnostics*), 150

## W

white() (*in module spreg.diagnostics*), 147